

Production Beyond the Firm: The Aggregate Value of Subcontracting*

Juan Segura[†]
CEMFI

This version: October 27, 2026

[Click here for the latest version](#)

Abstract

This paper studies the macroeconomic effects of subcontracting, the practice of delegating parts of production to external firms. Using administrative data on Portuguese firms, I show that a large share of aggregate input spending goes to subcontracting. Larger firms are more likely to subcontract, and those that subcontract do so more intensively. Subcontracting is unique among inputs in how much it moves with output: its expenses have an elasticity of 1.4 with respect to firm-level sales. I build a model of firm dynamics with labor adjustment frictions and task delegation. Firms subcontract for two reasons: flexibility, because delegating production allows them to circumvent adjustment frictions, and specialization, because the subcontractor is more efficient at performing certain tasks. Subcontracting raises aggregate productivity by 3.9%. Flexibility accounts for a fifth of this gain, by letting firms' size track their productivity more closely, and specialization for the remainder. Ignoring subcontracting overstates the productivity losses from firing costs by a sixth, and their output losses by two fifths. Moreover, subcontracting allows firms to absorb idiosyncratic shocks, but its flexibility largely vanishes under aggregate ones.

Keywords: Subcontracting, Adjustment costs, Firm dynamics, Aggregate productivity.

JEL Classification: D24, D25, E23, L23.

*This paper greatly benefited from conversations with Josep Pijoan-Mas and helpful comments from Samuel Bentolila, Gian Luca Clementi, Sebastian Fanelli, Nezih Guner, Hugo Hopenhayn, Federico Kochen, Pau Roldán-Blanco and seminar participants at CEMFI. I thank Pedro Raposo for his help with the data access process. I acknowledge financial support from Grant PRE2022-102243 funded by MCIN/AEI/ 10.13039/501100011033 and by “ESF +”, the Maria de Maeztu Unit of Excellence CEMFI MDM-2016-0684 and CEMFI CEX2020-001104-M, funded by MCIN/AEI/10.13039/501100011033, and CEMFI.

[†]CEMFI address: Casado del Alisal 5, 28014 Madrid, Spain. Email: juan.segura@cemfi.edu.es.

1 Introduction

Firms decide how much to produce and which parts of production to handle themselves. They may perform some tasks in-house and pay other firms to carry out others, a practice known as subcontracting. Two motives in particular make subcontracting valuable. First, specialized subcontractors can carry out some tasks more efficiently than the firms hiring them. Second, delegating tasks allows firms to change their scale without adjusting their in-house inputs. These two motives have different aggregate implications. Specialization raises productivity in the tasks that subcontractors perform better, so its aggregate effect depends on how many of those tasks firms delegate. Flexibility is valuable when firms need to change their scale and adjusting in-house inputs is costly, so its aggregate effect depends on how often firms face that situation. This paper addresses two questions. First, how much subcontracting contributes to aggregate productivity, and through which motive. Second, how accounting for subcontracting changes the measured consequences of labor adjustment frictions.

Subcontracting is a large and distinct input, and it raises aggregate productivity substantially. In the data, it accounts for 12% of firms' total variable costs, and its share rises steeply with firm size.¹ Moreover, it is the only input whose expenditure moves more than one-for-one with sales. To quantify its aggregate effects, I develop a model of firm dynamics in which adjusting in-house inputs is costly and firms can delegate the tasks that make up their production. Specialization arises because a subcontractor performs some tasks better than the firm, so delegating them raises efficiency. Flexibility arises because delegated tasks carry no in-house adjustment costs, so firms can change their scale through subcontracting. The calibrated model implies that subcontracting raises aggregate productivity by 3.9% and output by 9.8%. Despite the strong comovement of subcontracting with sales, flexibility accounts for only 19% of the productivity gain, and specialization for the remaining 81%. Ignoring subcontracting overstates the productivity losses from firing costs by a sixth, and their output losses by two fifths. Moreover, the flexibility of subcontracting lets firms absorb idiosyncratic shocks, but largely vanishes under aggregate ones.

I study subcontracting in Portugal, where administrative data report these expenses for all firms. The data cover firms' balance sheets, with yearly expenditures on traditional production inputs as well as on narrowly defined intermediates, allowing me to analyze subcontracting across and within firms. I first examine how variable costs are split across inputs along the size distribution. Firms at the top of the sales distribution spend propor-

¹Subcontracting records payments to other firms for carrying out tasks of the firm's own production process, rather than for materials or auxiliary services. Section 2.2 describes the input expenditure variables in detail.

tionally more on intermediates and less on labor than those at the bottom, and this gradient is primarily driven by subcontracting. In particular, 70% of firms in the top sales decile subcontract, against 21% in the bottom one; those that subcontract devote 20% and 9% of their variable costs to it, respectively. Firms in the top decile account for 61% of total sales.² These patterns hold both within and across firms.

I document evidence suggesting that subcontracting is used as a margin of adjustment to changing business conditions. First, among all production inputs, subcontracting is the one that tracks sales most closely. A 10% change in sales is associated with a 14% change in subcontracting expenditures in the same direction, while expenses on labor and capital, inputs typically associated with adjustment costs, move by less than 4%. The gap narrows at longer horizons, which give more weight to persistent changes in sales. Second, firm-level sales volatility correlates positively with subcontracting expenditure shares. The relationship holds across the size distribution and is stronger for larger firms, while labor cost shares display the opposite pattern. Taken together, these patterns point to subcontracting as a flexible margin for adjusting production scale, one that substitutes for costly adjustments to in-house inputs.

I construct a model of firm dynamics, labor adjustment frictions, and task delegation to rationalize these facts and quantify the aggregate implications of subcontracting. Production requires the completion of a continuum of sequential tasks, a segment of which can be partially or fully subcontracted. Labor adjustment is frictional, so delegating tasks to a subcontractor provides a flexible margin of adjustment. In addition, tasks differ in their codifiability, that is, in how precisely they can be specified to an outside firm. A specialized subcontractor performs highly codifiable tasks better than the firm, but not those with low codifiability.³ Firms also differ in how much of their production can be delegated, and those that can delegate more gain more from both motives. Flexibility and specialization capture the two reasons why firms in the model want to subcontract.

In equilibrium, firms delegate their most codifiable tasks first, and continue down the ranking while the subcontractor's efficiency justifies its relative price. When the shadow cost of in-house labor is high, as in periods of expansion, firms delegate beyond the tasks the subcontractor is better at: they accept efficiency losses in exchange for adjusting scale

²At a broader geographic scale, subcontracted production among manufacturing and mining firms within the economic territory of the EU countries amounts to 2.6% of the production value and 8.1% of the production volume. See the [Eurostat website](#) for details.

³Survey research on labor outsourcing cites flexibility, specialization and cost savings as the main reasons why firms engage in this practice ([Abraham and Taylor, 1996](#)). These channels apply naturally to the outsourcing of production activities more generally, which is the object of this paper. Motivated by this evidence, I model specialization gains as a reduced-form way to capture anything other than flexibility.

cheaply. When the shadow cost is low, as when firms hold excess labor, the opposite happens: firms retain tasks the subcontractor performs better rather than release workers. Firms thus trade specialization for flexibility at the margin. Motivated by the evidence on the extensive margin of subcontracting, I assume that access to it is costly and frictional. In order to subcontract, a firm must exert labor effort to adapt its production process, with the probability of success increasing in effort. Once obtained, access may be exogenously lost, in which case the firm must exert labor effort again to regain it.

Flexibility gains emerge endogenously through a shock-pooling mechanism. Subcontracting demand at the firm level responds to idiosyncratic shocks, but these get diluted in the aggregate, so the demand faced by the representative subcontractor is smoother than individual ones. The subcontractor operates in a competitive market, faces the same labor adjustment frictions as firms, and prices its services at its marginal cost of labor. Therefore, in a stationary equilibrium with constant aggregate subcontracting demand, the unit price of subcontracting equals the equilibrium wage. Subcontracting thus gives firms flexibility without weakening employment protection.

Subcontracting raises aggregate productivity by 3.9% and output by 9.8%. I obtain these numbers by comparing the calibrated economy with a counterfactual one where the subcontracting margin is removed and all other parameters are unchanged. Without subcontracting, firms can only change scale through in-house labor and pay the adjustment costs that come with it. They also forgo the efficiency gains of the tasks a subcontractor performs better. These are the two channels through which subcontracting adds to output.

Flexibility accounts for about a fifth of these gains, and specialization for the remaining four fifths. To isolate flexibility, I solve an economy in which the subcontractor is never more efficient than the firm: its efficiency gains at the most codifiable tasks are removed, while its losses at the least codifiable ones remain. In this economy, subcontracting is worth only what it saves in adjustment costs. Firms with a high shadow cost of labor still delegate low-codifiability tasks, accepting the efficiency losses in exchange for adjusting scale cheaply. To isolate specialization, I solve an economy in which access to subcontracting directly delivers the efficiency gains of codifiable tasks, but all tasks must be performed in-house and are therefore subject to in-house adjustment frictions. In this economy, delegating raises efficiency but does not make adjusting scale any cheaper. I split the gains between the two motives with a Shapley decomposition. Flexibility raises aggregate productivity by 0.73 percentage points, through the covariance between productivity and size. Delegating lets firms track their shocks more closely, so their scale moves more with their productivity, which reduces dynamic misallocation. Specialization gains, in turn, work like a productivity shifter for the firms that delegate, and raise aggregate productivity by 3.20 percentage

points.

Ignoring subcontracting overstates the productivity and output losses from firing costs, and gets the source of the productivity loss wrong. To show this, I remove firing costs in the baseline economy and in an economy without subcontracting, recalibrated to the same data as a researcher who overlooks this margin would do. Without a flexible margin of adjustment, the recalibrated economy needs larger transitory shocks and lower adjustment costs to match the data. In the baseline economy, removing firing costs raises productivity by 1.32%, mostly through selection, as more of the least productive firms exit. In the recalibrated economy, productivity rises by 1.52%, and the gain comes instead from the covariance between productivity and size, as firms track their shocks more closely. Output rises by more, 5.4% in the baseline economy and 7.7% in the recalibrated one, because inputs also respond through general equilibrium effects. In both, labor demand moves closer to its frictionless level, but less so in the baseline economy, where subcontracted labor carries no distortion. Measuring the costs of adjustment frictions thus requires accounting for the margins firms use to circumvent them.

In the model, subcontracting lets firms absorb idiosyncratic shocks, but provides little flexibility against aggregate ones. I apply the same persistent productivity shock to a single firm and to all firms at once, in the baseline and in the recalibrated economy. Firms respond almost equally to the aggregate shock in both economies, but much more to the idiosyncratic one in the baseline. On impact, the response of output to the idiosyncratic shock is 2.3 times that to the aggregate one in the baseline, against 1.7 in the recalibrated economy. Under an idiosyncratic shock, firms with access expand largely via subcontracting. An aggregate shock, instead, raises the wage, and the price of subcontracting even more, as the subcontractor must adjust its own workforce to serve all its clients at once and passes this cost on to them. As a result, firms with access lose most of their advantage, and the flexibility of subcontracting fades precisely when the whole economy needs to adjust.

Related literature. This paper contributes to several strands of the literature. The first is the literature on the aggregate costs of input adjustment frictions and firm dynamics (Hopenhayn and Rogerson, 1993; Cooper and Haltiwanger, 2006; Asker, Collard-Wexler and De Loecker, 2014; Roys, 2016; Boar, Gorea and Midrigan, 2025). These papers have established that frictional adjustment of inputs shapes how firms respond to their shocks, and can generate sizable dispersion in marginal revenue products and aggregate output losses. In contrast to their frameworks, which assume firms can only scale production in-house, I study subcontracting as a margin firms can use to circumvent such frictions. Accounting for this margin matters for measuring their costs, as ignoring it overstates the

productivity and output losses from firing costs. Moreover, subcontracting lets firms absorb idiosyncratic shocks, but hardly aggregate ones, so its value depends on the shocks firms face.

Second, a growing body of papers studies how firms combine intermediates and primary inputs (Huneus, Kroft and Lim, 2021; Dhyne, Petrin, Smeets and Warzynski, 2024; Hubmer, Chan, Ozkan, Salgado and Hong, 2025). Within this strand, the closest to my paper is Mertens and Schoefer (2024), who document that growing manufacturing firms shift from labor to intermediates, and rationalize it with an elasticity of substitution above one and a rising shadow price of labor, most likely due to monopsony. Using administrative data on firms across sectors, I show that this shift is largely driven by subcontracting, which is the input most responsive to sales and takes a larger share of costs in firms with more volatile sales. Structurally, I endogenize the substitutability between in-house and delegated production through the codifiability of tasks, and the shadow cost of labor rises because of adjustment frictions, which is why subcontracting responds most to transitory fluctuations.

The third is the literature on outsourcing and the boundaries of the firm, which I broadly group into the trade and network branch (Grossman and Helpman, 2002; Antràs, 2003; Antras and Helpman, 2004; Boehm and Oberfield, 2020; Rubbo, 2023; Baqaee, Burstein, Duprez and Farhi, 2025) and the labor branch (Goldschmidt and Schmieder, 2017; Bilal and Lhuillier, 2021). I contribute a dynamic model in which firms delegate tasks for two motives, specialization and flexibility. Delegation lowers a firm's unit cost of production, and makes this cost less sensitive to the shadow cost of in-house labor. Disciplined by how subcontracting and labor move with sales within firms, the model allows me to measure how much each motive contributes to the aggregate gains from subcontracting. Specialization accounts for most of these gains, even though subcontracting is the input most responsive to sales. This complements Arnarson, Buus, Moxnes, Munch and Xiang (2026), who find that specialization through task outsourcing in global value chains is central to firm performance.

Finally, the mechanisms of this paper relate to the literature on flexible labor arrangements, such as temporary contracts and staffing (Bentolila and Saint-Paul, 1992; Cahuc and Postel-Vinay, 2002; Caggese and Cuñat, 2008; Cahuc, Charlot and Malherbet, 2016; Autor, 2003; De Leon, Macaluso and Yeh, 2024). Subcontracting shares their flexibility role, but most of its aggregate productivity gain comes from specialization. The share of fixed-term contracts rises with size within firms but falls between firms, which Pijoan-Mas and Roldan-Blanco (2025) rationalize with a loss of firm-specific human capital. The subcontracting share rises with size along both dimensions, and in the model the between-firm pattern comes from delegation gains that are larger for firms whose production is more codifiable. The closest paper to mine is Bostanci (2026), who shows that stronger protection of trade

secrets increases labor outsourcing, and quantifies the resulting output gains in a model of industry dynamics with firing costs.⁴ Both papers arrive at similar task-based technologies. With the facts documented above, the model in this paper separates the two motives behind subcontracting and measures the consequences of labor adjustment frictions when firms can circumvent them.

Outline. The rest of the paper is structured as follows. Section 2 describes the data used in the paper and presents the empirical findings, organized in two blocks of empirical facts. Section 3 introduces the model of firm dynamics with adjustment frictions and task delegation. Section 4 discusses its calibration and fit, and the determinants of firm size in the calibrated economy. Section 5 quantifies the aggregate value of subcontracting. Section 6 studies how accounting for subcontracting changes the measured costs of labor adjustment frictions and the responses to idiosyncratic and aggregate shocks. Section 7 concludes.

2 Descriptive Evidence

This section lays out the empirical foundations of the paper. I begin by describing the data used throughout the analysis, followed by a characterization of the firm sample. I then document a set of empirical facts on subcontracting that motivate the theoretical framework developed in Section 3.

2.1 Portuguese Balance Sheet Data

The data source for this paper is *Sistema de Contas Integradas das Empresas* (SCIE), from which I obtain all information on input expenditures and sales. SCIE is collected on an annual basis by the Autoridade Tributária e Aduaneira and the Instituto Nacional de Estatística (INE)⁵ and covers the universe of non-financial, non-insurance, and non-public sector firms operating in Portugal, including companies, sole proprietors, and self-employed workers. The dataset is administrative in nature, as filing is mandatory for tax purposes, which ensures near-complete coverage and minimizes attrition bias relative to survey-based alternatives.

⁴His measure of outsourcing covers temporary staff and professional, technical, computer and advertising services, which differ from subcontracting.

⁵The Portuguese Tax Authority and the Portuguese National Statistics Institute, respectively.

Table 1: Summary statistics.

	Mean	SD	p10	p25	p50	p75	p90
Sales	475	1,352	24	51	121	323	944
Profit	25	165	−32	−7	5	26	84
Profit share	5.2	21.0	−8.3	0.4	5.5	12.2	21.5
Labor cost	143	363	10	19	44	112	300
Capital cost	19	67	0	1	3	11	37
Intermediate inputs	288	958	10	21	57	171	539
Materials	114	525	0	0	0	37	181
Subcontracting	56	382	0	0	0	8	66
Employment	8	18	1	2	3	7	17
Exp. share: labor	31.8	19.0	11.0	17.6	27.6	41.8	59.9
Exp. share: capital	4.3	5.6	0.4	1.0	2.6	5.4	9.7
Exp. share: intermediates	63.9	19.5	34.8	52.5	67.6	78.8	86.3
Exp. share: materials	25.2	24.7	0.0	0.0	21.0	45.9	61.3
Exp. share: subcontracting	12.4	18.8	0.0	0.0	2.8	17.6	41.3

Notes: Statistics computed for 1,758,186 firm-year observations. Ratios are weighted by the denominator variable. Monetary variables are expressed in thousands of 2022 EUR. Expenditure shares are expressed in percentages.

Although SCIE is available from 2004, detailed information on the input composition of firms is only available from 2010 onward, following a revision of variable definitions to comply with international accounting standards.⁶ The sample used in this paper therefore spans the period 2010–2022.

2.2 Variable Definition and Descriptive Statistics

The measure of output, py_{it} , for firm i in year t is sales of goods and services net of goods resold without transformation. Labor costs, wl_{it} , are computed as total personnel expenses, which include salaries, employer social security contributions, post-employment benefits, severance payments, workplace accident insurance, and other staff-related expenditures. Capital stocks are obtained via the perpetual inventory method, separately for tangible and intangible capital, and capital expenses, k_{it} , are computed as each stock times the sum of the interest rate and its depreciation rate (see Appendix A.2 for more details). Intermediate input expenses are the sum of seven categories: materials, subcontracting,

⁶The *Sistema de Normalização Contabilística* (SNC) replaced the previous *Plano Oficial de Contabilidade* in 2010.

specialized services, tools, energy, transportation, and other services.⁷ Materials, mat_{it} , capture the cost of goods and raw materials consumed in the firm’s own production process. Subcontracting, $subc_{it}$, records payments to other firms for autonomously carrying out tasks that are necessary for the firm’s own production process. The subcontracting fee covers the cost of the transformation service, including any factors the subcontractor employs, rather than the purchase of inputs. This distinguishes it from materials, which the firm transforms itself. It also excludes temporary staffing arrangements, where workers operate under the contracting firm’s direction using its own equipment, which are classified under specialized and other services. The remaining categories cover specialized technical services and other operational expenses, such as tools, energy, transportation, rents, insurance, and communications. Total variable costs, $cost_{it}$, are computed as the sum of capital, labor and intermediate input expenses. Profits, π_{it} , are the difference between output and total variable costs. I deflate nominal variables using the CPI deflator and set 2022 as the base year. Details on sample selection criteria are provided in Appendix A.3.

Summary statistics. Table 1 reveals substantial heterogeneity in firm size and input use. The average firm generates €475,000 in sales and €25,000 in profits, with both distributions being highly right-skewed, and the sales-weighted profit share ranging from −8.3% at the 10th percentile to 21.5% at the 90th. Intermediates are the dominant input, representing 63.9% of aggregate expenditures, followed by labor at 31.8%, while capital accounts for just 4.3%. The sample is dominated by small enterprises, with a median of 3 employees. Input use also exhibits substantial extensive-margin variation: both materials and subcontracting have median values of zero, and just 44.9% of firms report strictly positive subcontracting expenditures.

2.3 Subcontracting and Firm Size

This subsection uses the panel of Portuguese firms to document that the share of subcontracting in total variable costs increases with firm size, and decomposes this relationship into extensive and intensive margin components.

Larger firms spend proportionally more on subcontracting services. I first document that firms with higher sales devote a larger share of their input expenditures to intermediates, and that part of this pattern is attributable to subcontracting. I regress the share of input

⁷A detailed description of each intermediate input category based on definitions from the Portuguese accounting system is provided in Appendix A.1.

x in total variable costs, $\theta_{it}^x \equiv x_{it}/\text{cost}_{it}$, on the set of regressors $\{S_{it}^p\}_{p=1}^{100}$ and sector-year fixed effects $\gamma_{c(i)t}$:

$$\theta_{it}^x = \gamma_{c(i)t} + \sum_{p=1}^{100} \beta_p S_{it}^p + \epsilon_{it}, \quad (1)$$

where i denotes a firm and t a year.⁸ The regressor S_{it}^p is a dummy for the sales level of a firm falling into the within-industry-year p -th percentile in year t , and captures the relationship between firm size and input costs non-parametrically.

Panel (a) of Figure 1 shows the estimates from equation (1) for capital, labor and intermediate input expenses. The pattern is clear: firms at higher sales percentiles present larger shares of spending on intermediates, while cost shares of labor and capital are lower. This relationship is monotonic across the entire sales distribution. On average, the intermediate input cost share of a firm at the top of the sales distribution is 16 p.p. higher than that at the median sales level.

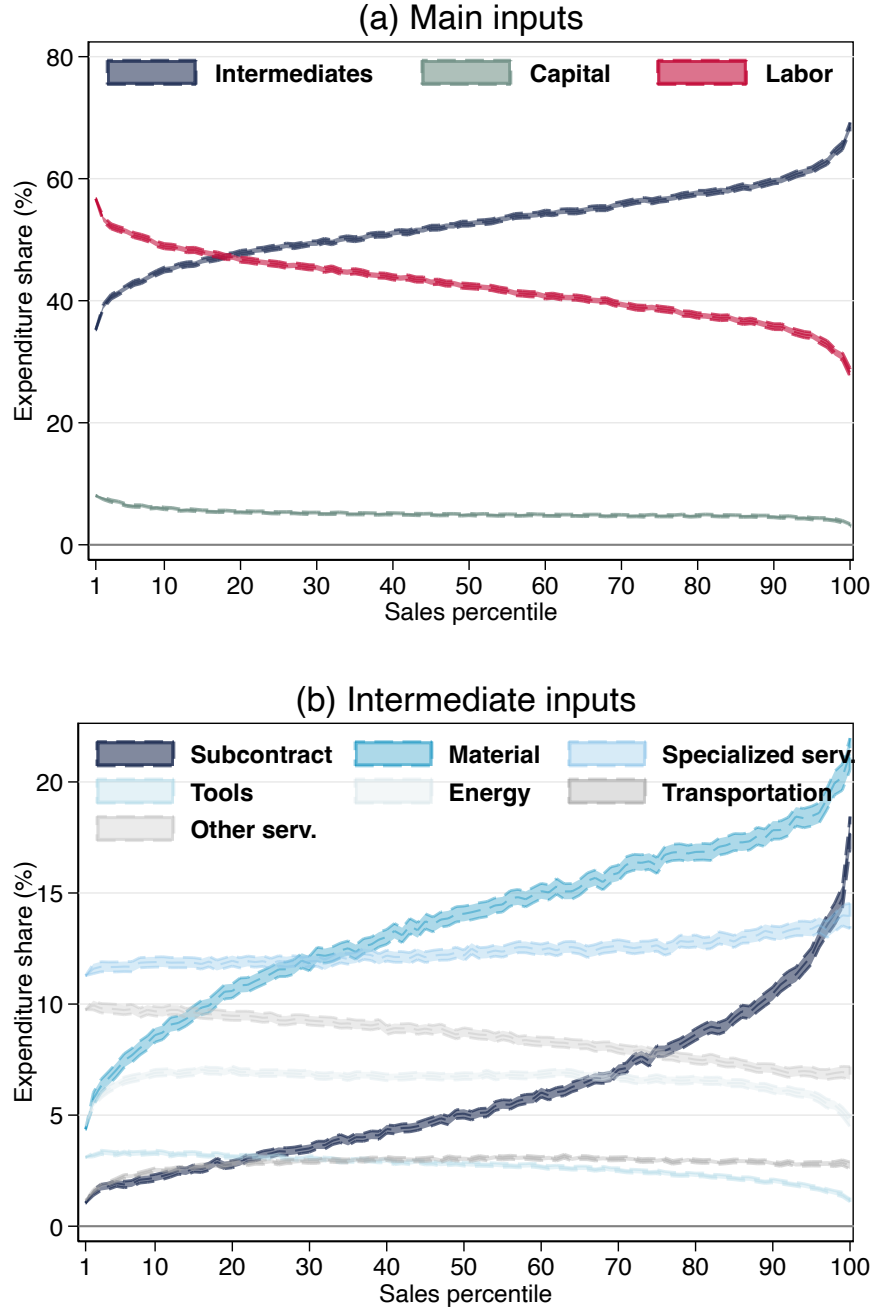
I next evaluate which intermediate inputs drive the pattern just described. Panel (b) of Figure 1 shows the estimates from equation (1) for all the intermediate input accounts described in Section 2.2. The cross-sectional positive relationship between intermediate input expenditures and sales is largely driven by two accounts: materials and subcontracting. The subcontracting expenditure share for a firm located at the top of the distribution is 17 p.p. larger than at the bottom, against 16 for materials. This relationship is steeper for materials at lower sales percentiles, but more pronounced for subcontracting services toward the top of the sales distribution. In contrast, the shares of the other five accounts change little with size, by at most about 3 p.p. between the bottom and the top of the distribution.

The increasing pattern in subcontracting is driven by both within-firm dynamics and between-firm differences. To capture the former, I replace the sector-year fixed effects in regression (1) with firm and year fixed effects; to compute the latter, I swap the sales percentile regressors for the within-firm average sales percentile and keep the sector-year fixed effects. Figure A.2 in the Appendix plots the two profiles together with the baseline specification.⁹ All curves are monotonically increasing in size, but the within-firm relationship is the steepest. On average, a firm at the bottom of the distribution barely subcontracts, but devotes 20% of its expenditures to subcontracting when at the top. The

⁸Because some firms do not spend on all inputs, the coefficients from this regression capture both intensive and extensive margin responses.

⁹To partially address concerns about short-panel bias, I estimate the results for the subset of firms observed for at least five years.

Figure 1: Expenditure shares and firm size.



Notes: Predicted values from regression (1) for different firm inputs. The shaded areas correspond to 95% confidence intervals clustered at the firm. Shares are computed over total variable costs, which are the sum of capital, labor and intermediate input costs. Materials, subcontracting, specialized services, tools, energy, transportation and other services are all the subcategories that comprise intermediate inputs.

between-firm and cross-sectional curves are almost identical, with the former being mildly flatter. Indeed, a simple decomposition shows that 30% of the cross-sectional variation

in subcontracting shares is within-firm, with the rest being between-firm variation.¹⁰ Together, these facts suggest there are structural differences across firms explaining the subcontracting-size relationship, but that firms also make use of subcontracting to grow or decline in size.

To better understand the dynamics of subcontracting, I decompose the relationship between subcontracting expenditures and firm size into its extensive and intensive margins. Figure 2 presents the estimated coefficients from equation (1) for the empirical probability of subcontracting (extensive margin) and the expenditure share of subcontracting for the subset of subcontracting firms (intensive margin). Firms in the top sales decile account for 61% of total sales. Among them, 70% subcontract, against 21% at the bottom decile. The intensive margin also increases considerably with firm size: those that subcontract devote 20% and 9% of their variable costs to it, respectively. The extensive margin is thus the largest driver of the positive relationship between subcontracting expenses and firm size. Subcontracting activity is also highly persistent, as 85% of the firms that subcontract in a given year continue to do so the following year.

Figures A.1 and A.3 in the Appendix show that these results also hold within firms. Figures A.4–A.9 in the Appendix show that they are robust when estimated within 1-digit CAE codes,¹¹ and so are not uniquely driven by sectoral composition.

2.4 Subcontracting as an Adjustment Margin

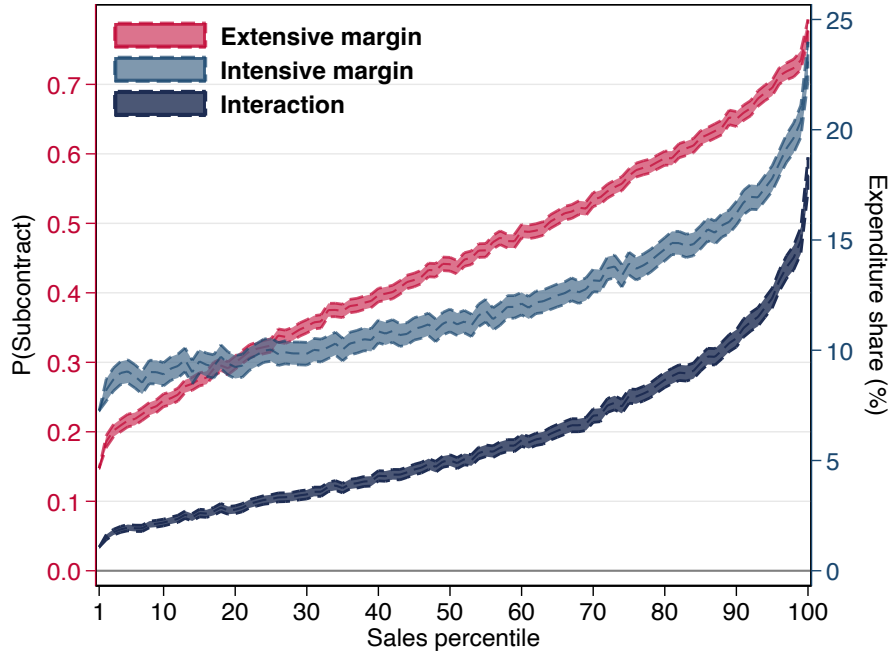
Having characterized the relationship between firm size and subcontracting, this subsection turns to the drivers of demand for subcontracting services. I present suggestive evidence that subcontracting serves as a flexible margin for accommodating demand fluctuations. Specifically, I report that subcontracting adjusts more readily than primary inputs to changes in sales, and that this difference is largest at short horizons. Then, I show that firms presenting greater sales volatility devote a larger share of total variable costs to subcontracting.

Subcontracting tracks output most closely among all inputs. Figure 3 presents estimates from regressions relating changes in the expenditures on each input to changes in sales. As discussed in Section 2.2, more than half of firm-year observations in the data do not subcontract. To cleanly estimate elasticities, I run the regressions only for firm-year

¹⁰This is the largest amount of within-firm variation among expenditure shares from the different inputs in the data.

¹¹Classificação Portuguesa das Actividades Económicas. CAE Rev. 3 closely resembles its European-level counterpart NACE.

Figure 2: Extensive and intensive margins of subcontracting.



Notes: Predicted values from regression (1) for the empirical probability of subcontracting, the expenditure share of subcontracting, and the product of the two estimates. The shaded areas correspond to 95% confidence intervals clustered at the firm. Confidence intervals for the interaction term are calculated using the Delta method.

observations that subcontract in both periods,¹² and for comparability use the same sample for the other inputs. To limit the influence of measurement error, the figure also excludes observations where the absolute change in log sales exceeds 0.5. For robustness, Tables A.3 and A.5 in the Appendix run the same exercise (except for subcontracting expenses) for all observations in the sample, and the results are essentially unchanged.

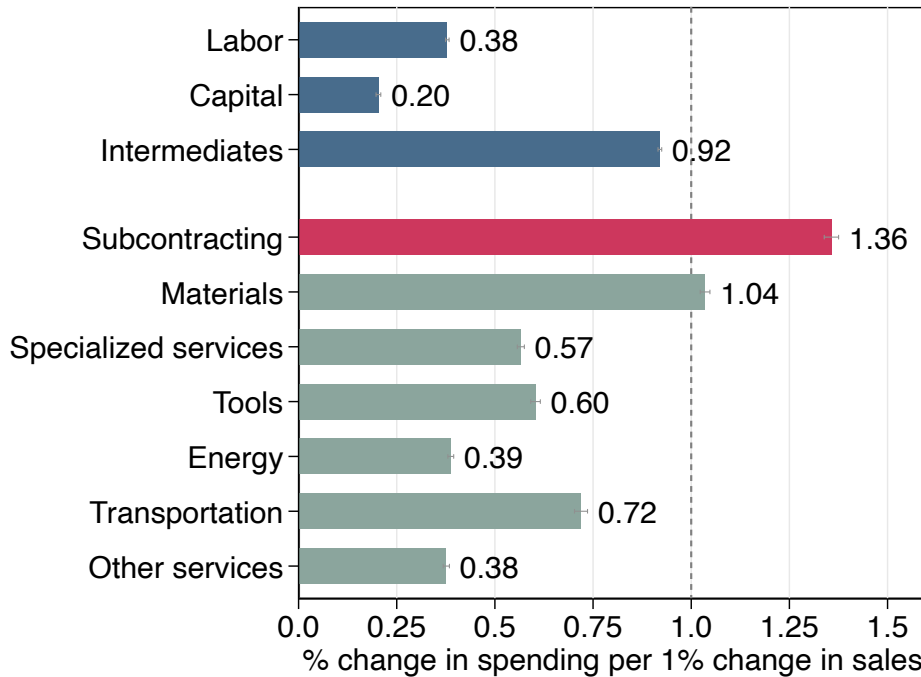
Labor and capital do not track sales closely. Among firms that subcontract, a 10% increase in sales is associated with a 3.8% increase in labor costs and a 2.0% increase in capital costs.¹³ Conversely, subcontracting is considerably more correlated with output. In particular, subcontracting expenditures rise more than one-for-one with sales: a 10% increase in output is associated with a 13.6% increase in subcontracting expenses. The figure extends the results to materials, another input considered more flexible.¹⁴ I find that the short-term comovement for materials is close to one.

¹²Chen and Roth (2024) warn against the use of “log-like” transformations for variables containing zeros.

¹³Donangelo, Gourio, Kehrig and Palacios (2019) and Boar et al. (2025) find similar patterns for US and Spanish firms, respectively.

¹⁴Because not all firms in the sample use materials for production, the comovements between materials and sales are estimated on a subsample that further conditions on materials expenditures being strictly positive in both years.

Figure 3: Comovement of input expenditures with sales.



Notes: Estimated coefficients from regressions of the annual log change in expenditures on each input on the annual log change in sales, with sector $\times$ year fixed effects. The sample includes firms with strictly positive subcontracting expenditures in both years and an absolute change in log sales below 0.5. For materials and the five other intermediate accounts, it further conditions on positive expenditures on that input in both years. Intermediates are the sum of the seven accounts below them. Error bars show 95% confidence intervals clustered at the firm. The dashed line marks a coefficient of one. Tables A.2 and A.4 in the Appendix report the estimates for all sales changes.

Without the restriction on sales changes, Table A.2 in the Appendix shows the same ranking. The coefficients of labor, capital and materials are very similar, and the one for subcontracting is lower, 1.13, but remains the highest and above one.

These results imply large heterogeneity in the comovement of different inputs with sales. Labor and capital, inputs historically associated with adjustment frictions, show very slow tracking of output. Materials present an elasticity of one, consistent with an input that is used in proportion to sales and not subject to costly adjustment. Other intermediate inputs, such as energy or transportation, comove with sales much less, with elasticities between 0.38 and 0.72. These include specialized and other services, the accounts that record temporary staffing, with elasticities of 0.57 and 0.38. Last, subcontracting moves more than one-for-one with sales, making it the most flexible adjustment margin among its users, and a potential offset to the sluggishness of labor and capital.

The comovement advantage of subcontracting is concentrated at shorter horizons.

Figure 4 reports the comovement coefficients re-estimated on k -year log changes, for $k = 1, \dots, 10$.¹⁵ Comparing the coefficients across horizons is informative about the types of shocks for which firms adjust through different inputs. Longer differences place more weight on persistent variation in sales: transitory variation enters only through the two years that are compared, while persistent variation accumulates over the window.¹⁶ The data point to the presence of both types of processes. Figure A.12 in the Appendix shows that the standard deviation of k -year log changes in sales increases with k , unlike under purely transitory fluctuations, but less than under an AR(1) process estimated on log sales.

Figure 4 shows that the coefficient for subcontracting moves from 1.36 at one year to 1.22 at ten years, while those for labor and capital rise steadily, from 0.38 and 0.20 to 0.71 and 0.67, respectively. The coefficients for labor and capital thus change much more across horizons than the one for subcontracting. The gap between subcontracting and in-house inputs narrows as variation in sales becomes more persistent: subcontracting absorbs relatively more of short-lived fluctuations, and labor and capital more of persistent ones. Table A.6 in the Appendix extends the exercise to all inputs. Subcontracting is the only one whose coefficient falls with the horizon, while materials stay close to one and the other inputs rise. Figure A.10 in the Appendix shows very similar profiles when the sample is restricted to firm-years observed in each of the ten previous years.

Sales volatility correlates positively with subcontracting expenditure shares. Figure 5 studies the relationship between input expenditure shares and sales volatility. Each line connects the estimated coefficients from a version of equation (1) with firm and year fixed effects. In particular, I regress the expenditure share for a given input on within-industry-year sales deciles for different quintiles of sales volatility. Volatility is defined as the standard deviation of the percentage change of sales over the five years prior to the present one.¹⁷

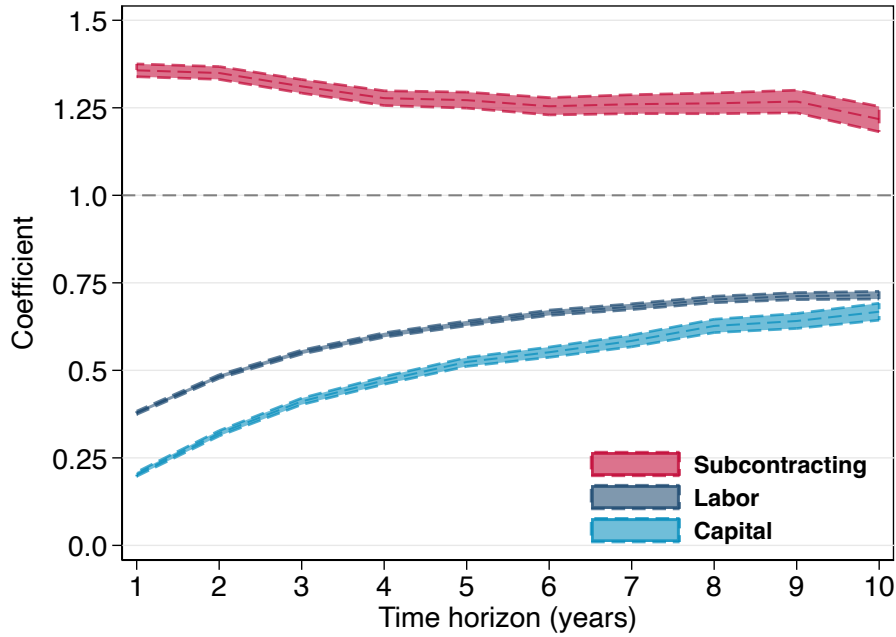
Panel (a) shows that firms with more volatile sales growth spend relatively more on subcontracting. The subcontracting expenditure share for a firm at the median sales level and top volatility quintile is 3 p.p. larger than at the bottom quintile. In line with the

¹⁵As in Figure 3, the sample includes only firms that subcontract in both periods. At horizon k , the sample excludes log changes in sales larger than $0.5 \text{sd}_k / \text{sd}_1$ in absolute value, where sd_k is the standard deviation of k -year log changes in sales.

¹⁶For a process comprising a persistent and a transitory component, $z_t = x_t + \varepsilon_t$ with $x_t = \rho x_{t-1} + \eta_t$ and ε_t i.i.d., the k -year change is $\Delta_k z_t = (x_t - x_{t-k}) + (\varepsilon_t - \varepsilon_{t-k})$, with variance $\text{Var}(\Delta_k z_t) = \frac{2\sigma_\eta^2}{1-\rho^2} (1 - \rho^k) + 2\sigma_\varepsilon^2$. The transitory term is constant across horizons, while the persistent one increases with k . The AR(1) benchmark in Figure A.12 in the Appendix sets $\sigma_\varepsilon = 0$, and the purely transitory benchmark sets $\sigma_\eta = 0$.

¹⁷Results are robust to longer time windows.

Figure 4: Input comovement with sales across horizons.

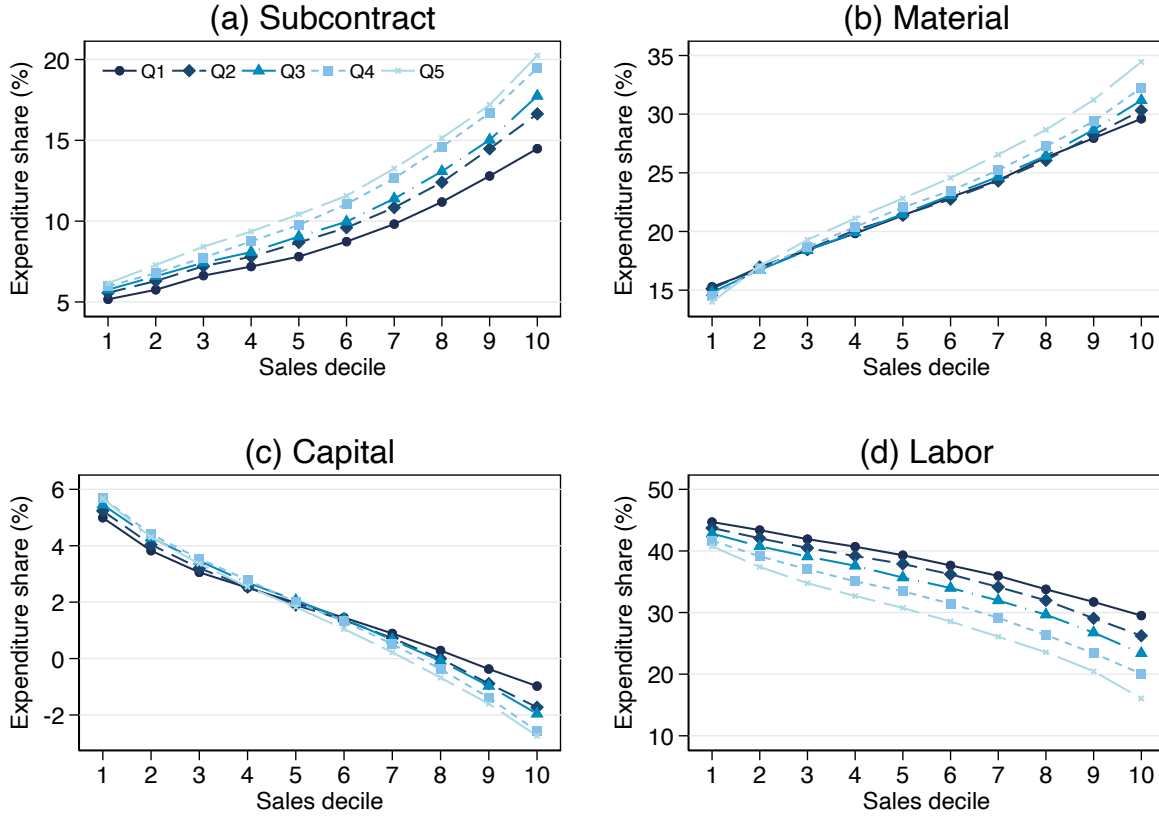


Notes: Estimated coefficients from regressions of k -year log changes in subcontracting expenditures, labor costs and capital costs on the k -year log change in sales, for $k = 1, \dots, 10$, among firms with positive subcontracting expenditures in both periods. The sample is restricted to log changes in sales below $0.5 \text{sd}_k / \text{sd}_1$ in absolute value, where sd_k is the standard deviation of k -year log changes in sales across all firms, so that the cutoff scales with the dispersion of sales changes at each horizon. Shaded areas correspond to 95% confidence intervals clustered at the firm. All regressions include sector $\times$ year fixed effects, are run at the firm level, and are unweighted.

findings from Section 2.3, subcontracting expenses are relatively larger for larger firms at all volatility levels, but the gradient is steeper in the higher volatility quintiles. Indeed, the same difference for firms at the top decile of the sales distribution is 6 p.p., twice as large as for firms at the median. Panel (b) shows similar patterns for materials: the expenditure share is greater for larger firms, and also larger in the higher volatility quintiles, though less so than for subcontracting. For firms in the top (middle) sales decile, being in the top volatility quintile is associated with an additional 5 p.p. (2 p.p.) in the materials expenditure share compared to the bottom quintile.

Panels (c) and (d) estimate the same specifications for capital and labor, the more sluggish inputs according to Figure 3. Both inputs' expenditure shares are decreasing in sales at all volatility levels, and decline more steeply at higher volatility quintiles. The pattern is particularly pronounced for labor. For firms at the top sales decile, being at the highest sales volatility quintile is associated with a labor expenditure share of 16%, while this share is 30% for firms at the bottom quintile. For capital expenditures, the difference is only 2 p.p. in the same direction.

Figure 5: Input usage and firm volatility.



Notes: This figure shows expenditure shares for different inputs by recent firm-level volatility. Coefficients estimated from regressing expenditure shares on within-industry-year sales decile dummies, with firm and year fixed effects, separately for each sales volatility quintile. Results displayed for firms observed for at least five years. Volatility is calculated as the standard deviation of the percentage change of sales over the last five periods: $Vol_{it} = \sqrt{\mathbb{V}_{t-5:t-1} \left(\frac{py_{it} - py_{it-1}}{py_{it-1}} \right)}$. Qx in the legend stands for the x-th quintile of the volatility measure distribution. The volatility measure is top and bottom trimmed at 1%. All coefficients are significant at the 5% level, with standard errors clustered by firm (not shown).

Figure A.11 in the Appendix repeats the exercise for total intermediates and the remaining intermediate accounts. The share of intermediates also rises with volatility, more so for larger firms, mostly driven by subcontracting and materials. Among the other accounts, only specialized services shows the same pattern, although less pronounced than subcontracting. The shares of tools, energy, transportation and other services do not.

These patterns admit two readings, both of which assign subcontracting the role of a flexible margin of adjustment. Under the first, firms facing more volatile demand substitute toward the inputs they can scale up and down at lower cost. Under the second, firms that rely on subcontracting accommodate shocks more fully, so a given sequence of shocks trans-

lates into larger realized fluctuations in sales.¹⁸ Under either interpretation, subcontracting works as a margin along which firms absorb variation in demand.

2.5 Taking Stock

The evidence assembled in this section establishes subcontracting as a quantitatively important and structurally distinct input. Its share of total variable costs increases monotonically with size, reaching 18% among the largest firms, and the increase operates through both the extensive and intensive margins. The pattern is driven by variation across firms and within firms. Subcontracting is also persistent: a firm that subcontracts in one year typically does so the next one.

The second set of facts concerns how subcontracting responds to changes in a firm's environment. Among all inputs, subcontracting expenditures track sales most closely, with elasticities above 1 among firms that subcontract, more than fully reflecting changes in sales. The gap with respect to labor's and capital's passthroughs is largest at short horizons, which place more weight on transitory variation in sales. Subcontracting is the only input whose comovement with sales falls with the horizon. Firms presenting more volatile past sales are also the ones devoting a larger share of expenditures to subcontracting, and a smaller share to labor and capital. In both its size profile and its comovement with sales, subcontracting also stands apart from other intermediate inputs, consistent with its connection to the production process rather than with auxiliary services. Together, these patterns are consistent with subcontracting offering firms a flexible margin through which to accommodate fluctuations in demand, operating alongside the gains from delegating tasks to more specialized producers, and reducing the cost of activities the firm could otherwise perform in-house. Separating these forces, and tracing their consequences for the allocation of resources across firms, requires a model of how and why firms choose to subcontract, which is the focus of the next section.

3 A Model of Subcontracting and Firm Dynamics

This section develops a model of firm dynamics with firing costs, extending [Hopenhayn and Rogerson \(1993\)](#) with a sequential production structure at the firm level, a subcontracting sector, hiring costs, and permanent differences across firms in how much of their

¹⁸Measuring volatility over a window up to $t - 1$ avoids overlap with the present expenditure share, but not this second channel if input mixes are persistent.

production they can delegate.¹⁹

3.1 Environment

Time is discrete. There is a mass of firms that evolves endogenously through entry and exit. Each entrant begins with zero workers and draws a productivity level and a permanent type from fixed entry distributions. Production requires transforming materials through the completion of a unit continuum of sequential tasks that require labor, a segment of which the firm can subcontract. The firm's type sets the size of this segment. Subcontracting services are offered by a representative subcontractor and accessing them is costly. The subcontractor performs the tasks demanded by firms at an efficiency that may be above or below the firm's own, and that varies across tasks; because in-house labor adjustment is frictional, subcontracting services also provide a flexible margin for adjusting production. The household side is deliberately simple.

Technology. An active firm i at time t must complete a continuum of sequential tasks to produce physical output g_{it} . Each stage $s \in [0, 1]$ transforms the intermediate input received from the preceding stage using labor $\ell_{it}(s)$, with materials m_{it} seeding the first stage:

$$d \ln q_{it}(s) / ds = b [\ln (A_{it}(s) \ell_{it}(s)) - \ln q_{it}(s)], \quad q_{it}(0) = m_{it}, \quad g_{it} = q_{it}(1),$$

where $A_{it}(s)$ is the stage productivity. Along the line, $b > 0$ is the rate at which labor used in stage s supersedes the labor embodied in the intermediate it receives. In the spirit of [Ciccone \(2002\)](#), integrating over the whole line and using the boundary conditions, output can be expressed as

$$\ln(g_{it}) = \alpha_m \ln(m_{it}) + \int_0^1 \zeta(s) \ln(A_{it}(s) \ell_{it}(s)) ds, \quad (2)$$

where $\alpha_m = e^{-b}$ is the output elasticity of materials, and $\zeta(s) = be^{-b(1-s)}$ is the output elasticity of stage- s labor.²⁰ Integrating elasticities over the continuum of tasks yields $\int_0^1 \zeta(s) ds = 1 - \alpha_m$, implying that output exhibits constant returns to scale in materials and labor. Due to the vertical structure of production, later stages carry larger weights $\zeta(s)$,

¹⁹Appendix B.1 presents a stylized version of the firm problem that isolates the flexibility role of subcontracting and reproduces some of the motivating empirical facts.

²⁰Appendix B.4 provides an alternative derivation of the production function (2) as the limit of N sequential tasks, each a Cobb–Douglas aggregator over the intermediate input it inherits and labor, with labor elasticity b/N .

meaning less of their work is superseded.

All tasks $s \in [0, \bar{s}_i]$, where $\bar{s}_i < 1$, are assumed to be ‘core’ tasks and must be produced in-house.²¹ In contrast, tasks belonging to the delegable segment $s \in (\bar{s}_i, 1]$ are ‘non-core’, and can be subcontracted. The boundary $\bar{s}_i$, and thus the size of the delegable segment, is a permanent firm type drawn at entry, $\bar{s}_i \sim F_{\bar{s}}(\bar{s})$.²² For tasks completed in-house, the firm uses its own labor $\ell_{H,it}(s)$ and stage productivity is the level $A_{it}(s) = A_0$; delegated tasks are carried out by a subcontractor using labor $\ell_{S,it}(s)$ and stage productivity $A_{it}(s) = A_0\phi\gamma(s; \bar{s}_i)$:

$$A_{it}(s) = \begin{cases} A_0 & \text{if } s \text{ is performed in-house at } t, \\ A_0\phi\gamma(s; \bar{s}_i) & \text{if } s \text{ is delegated at } t. \end{cases}$$

Beyond the level A_0 , common to all tasks, the stage productivity of delegated tasks comprises two factors. First, ϕ , common to all delegated tasks, captures the subcontractor’s overall efficiency. Second, the stage component $\gamma(s; \bar{s}_i)$ accounts for how codifiable different tasks are within the firm’s delegable segment, that is, how precisely they can be specified to an outside firm. Its dependence on the type comes only from the size of that segment: all firms share one profile, stretched over segments of different size. I refer to $\phi\gamma(s; \bar{s}_i)$, the subcontractor’s efficiency at a task relative to in-house production, as the task’s *yield*. It stands for any per-unit advantage of delegating the task, whether it arises from the subcontractor’s specialization or from a lower cost of performing the task outside the firm, and I call it specialization throughout.²³

I assume that $\gamma(s; \bar{s}_i)$ is strictly increasing in s : tasks closer to the end of the production line are easier to codify. Therefore, any delegated set is a terminal segment $(\hat{s}_{it}, 1]$.²⁴ A firm with access to subcontracting services thus chooses the boundary task $\hat{s}_{it} \in [\bar{s}_i, 1]$, together with its own labor $\ell_{H,it}(s)$ on tasks performed in-house, the subcontractor’s labor use $\ell_{S,it}(s)$ in delegated ones, and materials m_{it} . The firm completes in-house production up to task $\hat{s}_{it}$,

²¹I think of core tasks as those that require some firm-specific capability that cannot be replicated by outside producers, thus justifying the firm’s existence.

²²This type captures the ability to integrate outside work, or how codifiable the firm’s production process is overall.

²³Section 3.5 discusses an alternative interpretation where the same yields arise from coordination costs that rise with the number of tasks the firm performs in-house.

²⁴Monotonicity ensures the delegated set is a single segment, and the direction of the ranking is presentational. What matters for the firm is the set of yields available for delegation, not the position of the tasks carrying them: by Lemma 1, production depends on the delegated tasks only through their total weight and their ranked yields, both unchanged when the same yields are assigned to tasks in reverse order. Under a decreasing profile the firm delegates a segment adjacent to the core, $(\bar{s}_i, \hat{s}_{it}]$, and the intermediate input returns to the firm to be finished in-house.

hands over the intermediate input, and receives the finished output g_{it} . The revenue then becomes:

$$y_{it} = z_{it}\varepsilon_{it}g_{it}^\nu,$$

where $\nu < 1$ is the degree of decreasing returns to scale in revenue,²⁵ z_{it} is the persistent component of productivity, and ε_{it} its transitory component. Motivated by the evidence that sales fluctuations combine persistent and transitory variation (see Figure A.12 in the Appendix), I assume

$$\begin{aligned}\ln z_{it} &= \rho \ln z_{it-1} + \ln \eta_{it}, \\ \ln \eta_{it} &\sim \text{i.i.d. } \mathcal{N}(0, \sigma_\eta^2), \\ \ln \varepsilon_{it} &\sim \text{i.i.d. } \mathcal{N}(0, \sigma_\varepsilon^2).\end{aligned}\tag{3}$$

Labor adjustment is frictional. There are asymmetric adjustment costs on total in-house labor $l_{H,it} = \int_0^{\hat{s}_{it}} \ell_{H,it}(s)ds$, paid in units of the final good and proportional to the wage, $c(l_{H,it}, l_{H,it-1}) = w_t \left[\frac{\psi_H}{2} \mathbf{1}\{l_{H,it} > l_{H,it-1}\} + \frac{\psi_F}{2} \mathbf{1}\{l_{H,it} < l_{H,it-1}\} \right] \frac{(l_{H,it} - l_{H,it-1})^2}{l_{H,it-1}}$.²⁶ Upward adjustment costs are a reduced-form way of capturing search frictions, while downward adjustment costs represent firing costs.

Being an active firm requires paying a per-period fixed cost of operation κ_F . This cost is paid before knowing the current realization of productivity $(z_{it}, \varepsilon_{it})$. In addition, at the end of every period a firm is forced to exit with probability δ_X .

Task weights. In anticipation of Section 3.2, I introduce an alternative representation that simplifies both analytical derivations and their interpretation. Define the cumulative weight downstream of stage s :

$$u(s) = \int_s^1 \zeta(t)dt = 1 - e^{-b(1-s)},$$

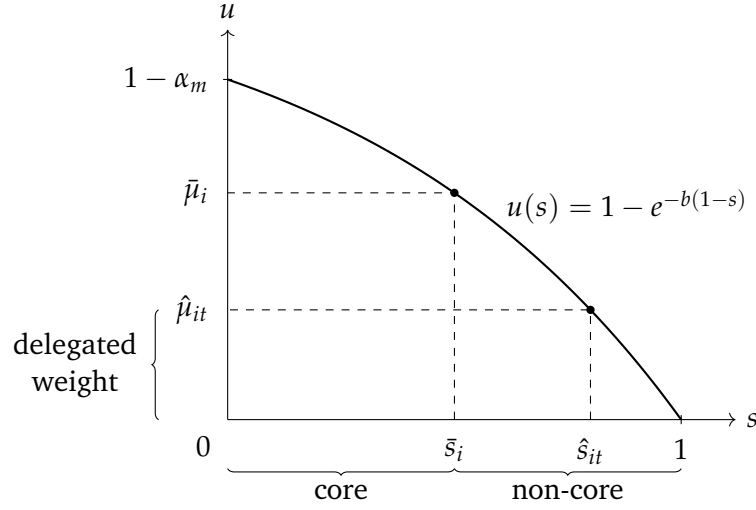
running from $1 - \alpha_m$ at the start of the line to 0 at its end, with inverse $s = 1 + \ln(1 - u)/b$.

Lemma 1. *In weight coordinates, the technology is Cobb–Douglas in materials and a uniform*

²⁵The final good $Y_t = \int y_{it}di$ serves as the numeraire, and is used for both consumption and materials.

²⁶For simplicity, it is assumed that labor is perfectly mobile across tasks.

Figure 6: From line positions to task weights.



Notes: The curve maps each stage s to the cumulative weight $u(s)$ of the tasks downstream. The core, non-core, and delegated segments map to the intervals drawn next to the axes.

continuum of labor tasks $u \in [0, 1 - \alpha_m]$, with a productivity shifter $\mathcal{K}$:

$$\ln(g_{it}) = \mathcal{K} + \alpha_m \ln(m_{it}) + \int_0^{1-\alpha_m} \ln(\tilde{A}_{it}(u) \tilde{\ell}_{it}(u)) du \quad (4)$$

$$\text{with } \mathcal{K} = \int_0^1 \zeta(s) \ln(\zeta(s)) ds,$$

where $\tilde{A}_{it}(u) = A_{it}(s(u))$, $\tilde{\ell}_{it}(u) = \ell_{it}(s(u))/\zeta(s(u))$, and, similarly for in-house and subcontracted labor, $\tilde{\ell}_{H,it}(u) = \ell_{H,it}(s(u))/\zeta(s(u))$ and $\tilde{\ell}_{S,it}(u) = \ell_{S,it}(s(u))/\zeta(s(u))$.

Proof. See Appendix B.5.

Corollary 1. Under weight coordinates $u(s)$:

- (i) The delegable segment $(\bar{s}_i, 1]$ maps to $[0, \bar{\mu}_i]$, where $\bar{\mu}_i = 1 - e^{-b(1-\bar{s}_i)}$; the core segment $[0, \bar{s}_i]$ maps to $[\bar{\mu}_i, 1 - \alpha_m]$.
- (ii) The delegated segment $(\hat{s}_{it}, 1]$ maps to $[0, \hat{\mu}_{it})$, where $\hat{\mu}_{it} = 1 - e^{-b(1-\hat{s}_{it})}$; choosing the boundary task $\hat{s}_{it} \in [\bar{s}_i, 1]$ is equivalent to choosing the delegated weight $\hat{\mu}_{it} \in [0, \bar{\mu}_i]$.
- (iii) The yield profile $\tilde{\gamma}(u; \bar{\mu}_i) = \gamma(s(u); \bar{s}_i)$ is strictly decreasing in u on $[0, \bar{\mu}_i]$.

Proof. See Appendix B.6.

The variable $u(s)$ measures the total output elasticity of the labor employed downstream of s . Lemma 1 shows that in these coordinates all tasks enter production symmetrically: the

weighting function $\zeta(s)$ disappears, and tasks differ only through their productivity $\tilde{A}_{it}(u)$, not through their position on the line. Corollary 1 restates the delegation decision and the yield profile under the new coordinates; Figure 6 illustrates the mapping. The firm's delegation problem thus reduces to choosing a single delegated weight $\hat{\mu}_{it} \in [0, \bar{\mu}_i]$. The stretch of the common profile is defined in weight coordinates: $\tilde{\gamma}(u; \bar{\mu}_i)$ depends on u and $\bar{\mu}_i$ only through $u/\bar{\mu}_i \in [0, 1]$. Last, I adopt the normalization $A_0 = e^{-\mathcal{K}/(1-\alpha_m)}$ from here onwards to remove the constant term from expression (4).²⁷

Subcontracting. Subcontracting services are supplied by a representative subcontractor in a perfectly competitive market. It completes the delegated tasks at the stage productivities introduced above, supplying the labor $\tilde{\ell}_{S,it}(u)$ demanded by each client. It hires this labor at wage w_t in the same market as firms, and is subject to the same adjustment cost function, applied to its aggregate labor demand. For a firm i delegating the tasks $[0, \hat{\mu}_{it})$, the subcontractor receives the intermediate input $\hat{q}_{it} \equiv q_{it}(\hat{s}_{it})$ and returns the finished output g_{it} . Applying the sequential technology of equation (2) from the handover point onwards and expressing it in weight coordinates,

$$\ln(g_{it}) = (1 - \hat{\mu}_{it}) \ln(\hat{q}_{it}) + \int_0^{\hat{\mu}_{it}} \ln(A_0 \phi \tilde{\gamma}(u; \bar{\mu}_i) \tilde{\ell}_{S,it}(u)) du + \mathcal{K}_q(\hat{\mu}_{it}),$$

where $\mathcal{K}_q(\mu) = \int_0^\mu \ln(\hat{\zeta}(u)) du$ and $\hat{\zeta}(u) = \zeta(s(u)) = b(1 - u)$.²⁸ Delegated tasks are performed at the firm's own stage productivity scaled by the factor $\phi \tilde{\gamma}(u; \bar{\mu}_i)$.

The subcontractor is paid for the transformation service it provides, not for the output it returns. It takes as given the price ω_t per unit of labor deployed, common across clients, so its revenue and costs are

$$R_{S,t} = \omega_t L_{S,t}, \quad C_{S,t} = w_t L_{S,t} + c(L_{S,t}, L_{S,t-1}), \quad L_{S,t} = \int_i \int_0^{\hat{\mu}_{it}} \tilde{\ell}_{S,it}(u) du di.$$

Adjustment costs on delegated labor are therefore borne by the subcontractor on its total position, not by each client separately: a firm that varies its delegated weight pays ω_t per unit of labor deployed and no adjustment cost on it. This is the sense in which subcontracting provides a flexible margin. How much of the subcontractor's adjustment cost is reflected in ω_t depends on the behavior of aggregate subcontracted labor.

Firms must adapt their technology before they can subcontract. A firm without access

²⁷The term A_0 enters revenue only through the common factor $A_0^{v(1-\alpha_m)}$, so fixing it is a normalization of productivity units, which are normalized during the calibration.

²⁸ $\mathcal{K}_q(\mu) = \mu \ln b - \mu - (1 - \mu) \ln(1 - \mu)$ depends on the delegated weight but not on the allocation of labor across tasks. The constant in Lemma 1 is the special case $\mathcal{K} = \mathcal{K}_q(1 - \alpha_m)$.

chooses labor effort $e \geq 0$, paid at the wage, and its technology becomes compatible with subcontracting with probability $p(e)$.²⁹ A firm with access faces an exogenous probability δ of losing it each period.³⁰ Adaptation and loss of access occur at the beginning of the period, after entry and exit and before the realization of productivity, so a firm that loses access cannot regain it within the same period.

Entry. There is an unbounded mass of ex ante identical potential entrants and, each period, a mass E_t of them enters. Upon paying κ_E , new firms draw a permanent type from $F_{\bar{s}}(\bar{s})$ and choose labor effort, so they may begin producing with subcontracting access. They then receive an initial productivity draw, with $\ln z \sim \mathcal{N}(\mu_E, \sigma_E^2)$, and ε drawn as in (3). Entrants choose their initial workforce without adjustment costs.

Preferences. The representative household maximizes lifetime utility

$$U = \sum_{t=0}^{\infty} \beta^t \left(C_t - \theta \frac{N_t^{1+1/\iota}}{1+1/\iota} \right),$$

where C_t denotes consumption of a single final good in period t , N_t is the labor supplied to the economy, β is the discount factor, ι is the Frisch elasticity of labor supply, and θ is the value of leisure relative to consumption. Every period, the household chooses how much to consume and how many units of labor to supply to firms and the subcontractor at equilibrium wage w_t . The representative household owns all technologies and the subcontracting sector, so all aggregate profits Π_t are rebated to her. Moreover, labor adjustment costs from firing, denoted T_t , are also rebated to the household.³¹ Thus, every period the budget constraint of the household is

$$C_t = w_t N_t + \Pi_t + T_t. \quad (5)$$

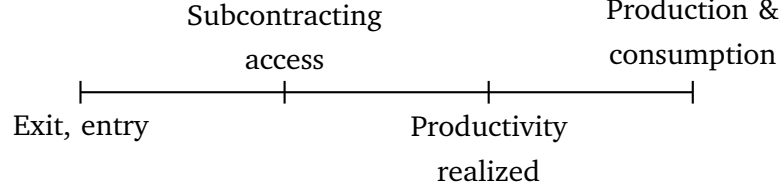
Timing. At the beginning of a period, firms that exit pay the firing cost $c(0, l_H)$ and potential entrants decide whether to join the economy by paying κ_E , upon which they draw their type. All active firms pay κ_F . Firms without subcontracting access, entrants included, then choose labor effort, and firms currently with access lose it with probability δ .

²⁹I assume the function $p(e)$ is differentiable, strictly increasing, and strictly concave, with $p(0) = 0$, $\lim_{e \rightarrow \infty} p(e) = 1$, $\lim_{e \rightarrow 0} p'(e) = \infty$, and $\lim_{e \rightarrow \infty} p'(e) = 0$.

³⁰I interpret loss of access as obsolescence of a firm's adaptation, making adaptation a recurring decision rather than a once-and-for-all one.

³¹Since firing costs are interpreted as severance payments, they are rebated to the household. Hiring costs are treated as deadweight losses because their interpretation is purely technological.

Immediately after, incumbents and entrants observe their productivity draws (z, ε) . Then, production takes place and the household consumes. Last, a fraction δ_X of firms is forced to exit, and the remaining firms decide whether to exit or continue operating next period given their current productivity, labor and access status.



3.2 Decision Rules

I next describe how the production choices of firms are affected by input adjustment frictions and subcontracting access. Later, I explain how agent responses and market clearing shape the economy's equilibrium.

Household decision. The problem of the household is static. Total labor supply is obtained by equating the marginal disutility of labor to its marginal benefit:

$$\theta N_t^{1/\iota} = w_t. \quad (6)$$

Given the absence of wealth effects, the amount of labor supplied by the household is pinned down by the wage and preference parameters. Aggregate consumption is obtained from the household's budget constraint defined in (5).

Subcontractor's problem and pricing. The subcontractor takes the price of the service ω_t as given and chooses labor $L_{S,t}$ to maximize the present value of profits, discounted at factor β :

$$W_t(L_{S,t-1}) = \max_{L_{S,t} \geq 0} (\omega_t - w_t)L_{S,t} - c(L_{S,t}, L_{S,t-1}) + \beta W_{t+1}(L_{S,t}).$$

The associated optimality condition defines the subcontractor's supply curve:

$$\omega_t = w_t + c_1(L_{S,t}, L_{S,t-1}) + \beta c_2(L_{S,t+1}, L_{S,t}), \quad (7)$$

where c_j is the derivative of c with respect to its j -th argument. Services are priced at the wage plus the marginal cost of adjusting the pooled workforce, current and expected. Equi-

librium quantities follow from the aggregate demand of clients. A firm delegating the weight $\hat{\mu}_{it}$ and requesting $\tilde{\ell}_{S,it}(u)$ units of labor at each delegated task pays $\omega_t \int_0^{\hat{\mu}_{it}} \tilde{\ell}_{S,it}(u) du$.

In a stationary equilibrium the measure of firms with access and their distribution over states are constant, so the subcontractor pools the idiosyncratic shocks of its clients and aggregate demand $L_{S,t}$ is also constant. Therefore, it never adjusts its workforce, so the service is priced at $\omega_t = w_t$, and the subcontractor breaks even. Delegated labor then carries no adjustment cost, unlike in-house labor, even though the two are subject to the same cost function and hired in the same market. This pooling is the source of the flexibility that subcontracting provides: a margin through which firms adjust output without incurring their own labor adjustment costs. Away from the steady state, aggregate subcontracted labor moves, and (7) prices the resulting adjustment into ω_t .³² The flexibility just described and the potential efficiency advantages captured by the yields $\phi\tilde{\gamma}(u; \bar{\mu}_i)$ are the two benefits of subcontracting relative to in-house production.

Incumbents and production. An incumbent firm chooses materials m_{it} and the labor deployed at each task it performs $\tilde{\ell}_{H,it}(u)$. If the firm has access to subcontracting, it also chooses the delegated weight $\hat{\mu}_{it}$ and the subcontractor labor demanded at each delegated task $\tilde{\ell}_{S,it}(u)$.³³ The value of a firm depends on its productivity components $(z_{it}, \varepsilon_{it})$, last period's in-house labor $l_{H,it-1}$, its permanent type $\bar{\mu}_i$, and its subcontracting access status $a_{it} \in \{0, 1\}$.

Because tasks are symmetric in weight coordinates, labor is optimally spread evenly across them: given total in-house labor and delegated weight, $\tilde{\ell}_{H,it} = l_{H,it} / (1 - \alpha_m - \hat{\mu}_{it})$ on every retained task,³⁴ and the subcontractor deploys a common $\tilde{\ell}_{S,it} = l_{S,it} / \hat{\mu}_{it}$ on every delegated one.³⁵

³²The subcontractor is owned by the representative household, so its profits, which arise only outside of the steady state, are rebated as a lump sum.

³³Each task is performed either in-house or by the subcontractor, so the two labor densities have complementary supports: $\tilde{\ell}_{S,it}(u) > 0$ only for $u < \hat{\mu}_{it}$, and $\tilde{\ell}_{H,it}(u) > 0$ only for $u \geq \hat{\mu}_{it}$.

³⁴This reduction of task-level choices to scalars is one of the payoffs of working in weight coordinates u rather than physical coordinates s .

³⁵Appendix B.7 derives this result together with all optimality conditions in this section.

The problem of a firm without access can be written in recursive form as:³⁶

$$V_{NS}(z, \varepsilon, l_{H,-1}, \bar{\mu}) = \max_{l_H, m} \left\{ z\varepsilon \left[m^{\alpha_m} \left(\frac{l_H}{1 - \alpha_m} \right)^{1 - \alpha_m} \right]^v \right. \\ \left. - wl_H - c(l_H, l_{H,-1}) - m - \kappa_F \right. \\ \left. + \beta \left[(1 - \delta_X) \max \left\{ -c(0, l_H), \mathcal{V}_{NS}(z, l_H, \bar{\mu}) \right\} - \delta_X c(0, l_H) \right] \right\}. \quad (8)$$

The first line includes revenues from production. The second line captures labor-related costs, material expenses, and the fixed cost of operation. The third line presents the continuation value. At the end of the period, the firm is forced to exit with probability δ_X ; otherwise, it decides whether to exit or to continue operating. An exiting firm pays the cost of dismissing its workforce, $c(0, l_H)$, at the beginning of the next period.³⁷

The optimality conditions of a firm without access can be characterized by a small set of equations. First, the input mix satisfies

$$\frac{m}{l_H} = \frac{\alpha_m}{1 - \alpha_m} \tilde{w}, \quad (9)$$

where $\tilde{w} := w + c_1(l_H, l_{H,-1}) + \beta\Omega(z, l_H, \bar{\mu}, a)$ is the shadow cost of in-house labor and

$$\Omega(z, l_H, \bar{\mu}, a) = \delta_X c_2(0, l_H) + (1 - \delta_X) \left[\mathbf{1}\{\mathcal{V}(z, l_H, \bar{\mu}, a) \geq -c(0, l_H)\} \mathbb{E}[c_2(l_{H,+1}, l_H)] \right. \\ \left. + \mathbf{1}\{\mathcal{V}(z, l_H, \bar{\mu}, a) < -c(0, l_H)\} c_2(0, l_H) \right]$$

is the expected marginal effect of present labor choices on next period's adjustment cost. Labor adjustment frictions separate the shadow cost from the wage, tilting the input mix away from the frictionless benchmark. Second, labor is set so that its marginal revenue product equals its shadow cost, $(1 - \alpha_m)vy/l_H = \tilde{w}$.

³⁶I define $\mathcal{V}(\cdot, a = 1) = \mathcal{V}_S$ and $\mathcal{V}(\cdot, a = 0) = \mathcal{V}_{NS}$, and use the two notations interchangeably throughout the paper.

³⁷The Portuguese Labour Code grants the workers of a firm that closes the same compensation as in a collective dismissal. See the [Labour Code](#) for details.

When the firm has access to subcontracting, the problem becomes

$$V_S(z, \varepsilon, l_{H,-1}, \bar{\mu}) = \max_{l_H, m, \hat{\mu} \in [0, \bar{\mu}], l_S} \left\{ z \varepsilon B(\hat{\mu})^\nu \left[m^{\alpha_m} \left(\frac{\phi l_S}{\hat{\mu}} \right)^{\hat{\mu}} \left(\frac{l_H}{1 - \alpha_m - \hat{\mu}} \right)^{1 - \alpha_m - \hat{\mu}} \right]^\nu \right. \\ \left. - w l_H - c(l_H, l_{H,-1}) - \omega l_S - m - \kappa_F \right. \\ \left. + \beta \left[(1 - \delta_X) \max \left\{ -c(0, l_H), \mathcal{V}_S(z, l_H, \bar{\mu}) \right\} - \delta_X c(0, l_H) \right] \right\}, \quad (10)$$

where $B(\hat{\mu}) = \exp \left\{ \int_0^{\hat{\mu}} \ln \tilde{\gamma}(u; \bar{\mu}) du \right\}$ collects the stage components of the delegated tasks and acts as an endogenous productivity shifter. Relative to the problem without access in equation (8), the firm chooses two additional scalars: the delegated weight $\hat{\mu}$ and the total subcontracted labor l_S , paying the subcontracting price ω per unit. Taking the limit $\hat{\mu} \rightarrow 0$ recovers the period problem without access in (8).

Proposition 1. *The optimal delegated weight of a firm with access satisfies*

$$\phi \tilde{\gamma}(\hat{\mu}; \bar{\mu}) = \frac{\omega}{\tilde{w}} \quad (11)$$

whenever the implied $\hat{\mu}$ lies in $(0, \bar{\mu})$, and is at a corner otherwise.

Proof. See Appendix B.7.2.

The firm delegates until the marginal task's yield equals the price of subcontracting relative to the shadow cost of in-house labor. The condition moves with the firm's state: a high shadow cost lowers the required yield, pushing the margin into tasks the firm itself performs better ($\phi \tilde{\gamma}(u; \bar{\mu}) < 1$); a low shadow cost pulls it back, and the firm retains tasks the subcontractor holds an advantage in. Delegation thus responds to adjustment pressure, not only to the subcontractor's efficiency. At the margin, firms trade specialization for flexibility, giving up efficiency to avoid hiring when their labor is scarce and firing when it is trapped.

The cost of every input equals its weight in revenue:

$$m = \alpha_m \nu y, \quad \omega l_S = \hat{\mu} \nu y, \quad \tilde{w} l_H = (1 - \alpha_m - \hat{\mu}) \nu y. \quad (12)$$

Proposition 2. *At an interior optimum, the unit cost of production of a firm with subcontracting access is*

$$c_g = e^{-R(\hat{\mu})} \alpha_m^{-\alpha_m} \tilde{w}^{1 - \alpha_m}, \quad R(\hat{\mu}) = \int_0^{\hat{\mu}} \ln \frac{\tilde{\gamma}(u; \bar{\mu})}{\tilde{\gamma}(\hat{\mu}; \bar{\mu})} du \geq 0, \quad (13)$$

with $\partial \ln c_g / \partial \ln \tilde{w} = 1 - \alpha_m - \hat{\mu}$. At a corner, the same expression holds with $R(\hat{\mu})$ replaced by $R_c(\hat{\mu}) = \int_0^{\hat{\mu}} \ln(\phi \tilde{\gamma}(u; \bar{\mu}) \tilde{w} / \omega) du$.

Proof. See Appendix B.7.3.

Delegation lowers unit costs by a factor $e^{-R(\hat{\mu})}$. By Proposition 1, the marginal delegated task costs the same through subcontracting as in-house; every other delegated task has a higher yield and costs strictly less, saving $\ln(\tilde{\gamma}(u; \bar{\mu}) / \tilde{\gamma}(\hat{\mu}; \bar{\mu}))$ per unit of weight u . The term $R(\hat{\mu})$ represents the *delegation gains*: the cost saving relative to completing delegated tasks in-house. In addition, delegation reduces the exposure of costs to in-house labor. The elasticity of the unit cost to the shadow cost $\tilde{w}$ is $1 - \alpha_m - \hat{\mu}$, against $1 - \alpha_m$ for a firm producing everything in-house.

Delegation gains can be split into two components. Let $\hat{\mu}_\omega$ denote the delegated weight when $\tilde{w} = \omega$. Then

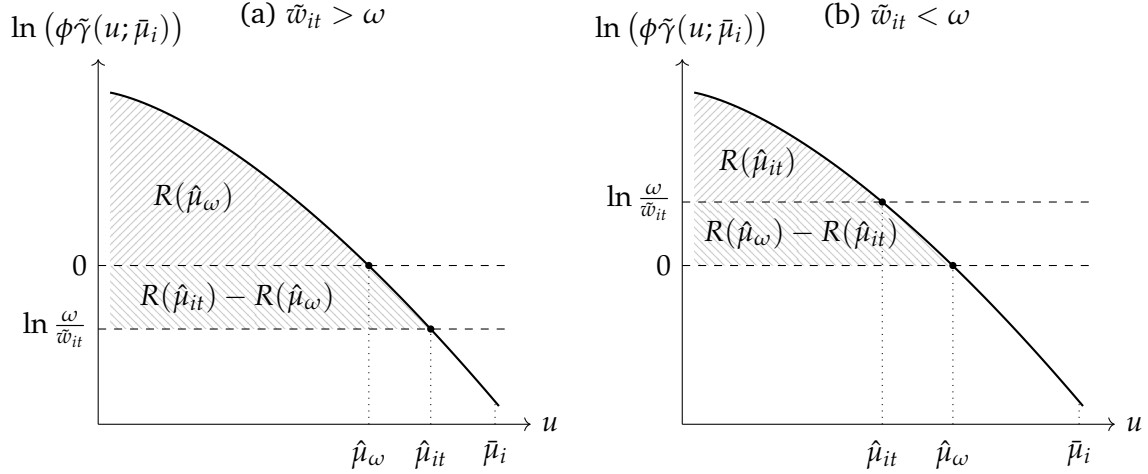
$$R(\hat{\mu}_{it}) = \underbrace{R(\hat{\mu}_\omega)}_{\text{permanent component}} + \underbrace{R(\hat{\mu}_{it}) - R(\hat{\mu}_\omega)}_{\text{state component}}. \quad (14)$$

The permanent component collects the gains when $\tilde{w} = \omega$; the state component moves realized gains around that level through in-house labor needs. Figure 7 illustrates them. In Panel (a), the state component is positive: when $\tilde{w}_{it} > \omega$, the firm delegates tasks the subcontractor completes less efficiently because completing them in-house would be even more expensive. In Panel (b), the state component is negative: when $\tilde{w}_{it} < \omega$, the firm retains tasks the subcontractor completes more efficiently, because using its inherited labor is cheaper.

These results map into the reasons why firms subcontract in the model. The permanent component reflects the specialization motive: gains available at the undistorted margin $\tilde{w}_{it} = \omega$, present even for a firm that never adjusts its scale. The flexibility gain comes from moving the margin itself: when $\tilde{w}_{it}$ is high, the firm expands effective input use without hiring; when $\tilde{w}_{it}$ is low, it cuts subcontracting rather than workers. The state component is the footprint of these adjustments on realized delegation gains, but not their value: the firm benefits from the margin in both cases.

A firm's problem depends on its access state. With access, the firm solves the problem in equation (10); without it, the firm is restricted to in-house production and solves equation (8). Since the limit $\hat{\mu} \rightarrow 0$ in the former recovers the latter's period problem, access weakly raises the flow payoff. I denote by $\chi \in \{0, 1\}$ the optimal exit decision embedded in the continuation values, with $\chi = 1$ implying firm exit. The two problems share the same structure and differ only through the delegation margin and their continuation values,

Figure 7: The delegation margin.



Notes: The decreasing curve is the log yield of task u , $\ln(\phi\tilde{\gamma}(u; \bar{\mu}_i))$. The horizontal lines are par, at zero, and the delegation threshold $\ln(\omega/\tilde{w}_{it})$. The cutoff $\hat{\mu}_{it}$ is the delegated weight, $\hat{\mu}_\omega$ denotes the delegated weight when $\tilde{w}_{it} = \omega$, and $\bar{\mu}_i$ is the delegable ceiling. The gains from delegation are the areas between the curve and the threshold over the delegated tasks. They equal $R(\cdot)$ in Proposition 2 at an interior optimum and $R_c(\cdot)$ at a corner. When $\tilde{w}_{it} = \omega$, the threshold is par.

which depend on the access state as described next.

Subcontracting access. A firm without access chooses its labor effort before observing its current productivity. The value of the adaptation option is

$$\mathcal{J}(z, l_{H,-1}, \bar{\mu}) = \max_{e \geq 0} \left\{ -we + p(e) \mathbb{E}[V_S(z', \varepsilon', l_{H,-1}, \bar{\mu}) - V_{NS}(z', \varepsilon', l_{H,-1}, \bar{\mu})] \right\}, \quad (15)$$

with first-order condition $w = p'(e^*) \mathbb{E}[V_S(z', \varepsilon', l_{H,-1}, \bar{\mu}) - V_{NS}(z', \varepsilon', l_{H,-1}, \bar{\mu})]$, where expectations are taken over the productivity process in equation (3). Optimal effort is increasing in the expected value of access. Altogether, the continuation value for a firm without access is

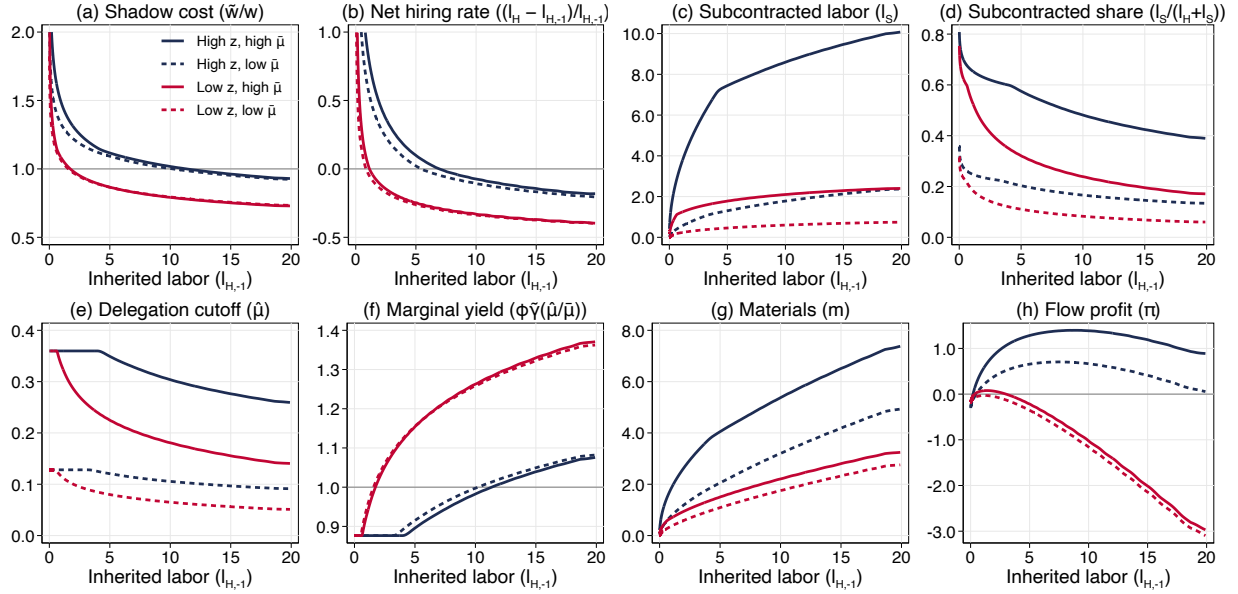
$$\mathcal{V}_{NS}(z, l_{H,-1}, \bar{\mu}) = \mathbb{E}[V_{NS}(z', \varepsilon', l_{H,-1}, \bar{\mu})] + \mathcal{J}(z, l_{H,-1}, \bar{\mu}).$$

The continuation value for a firm with access follows

$$\mathcal{V}_S(z, l_{H,-1}, \bar{\mu}) = (1 - \delta) \mathbb{E}[V_S(z', \varepsilon', l_{H,-1}, \bar{\mu})] + \delta \mathbb{E}[V_{NS}(z', \varepsilon', l_{H,-1}, \bar{\mu})].$$

With probability $1 - \delta$, access is preserved; with probability δ , access is lost and the firm enters next period without access.

Figure 8: Decision rules, firms with subcontracting access.

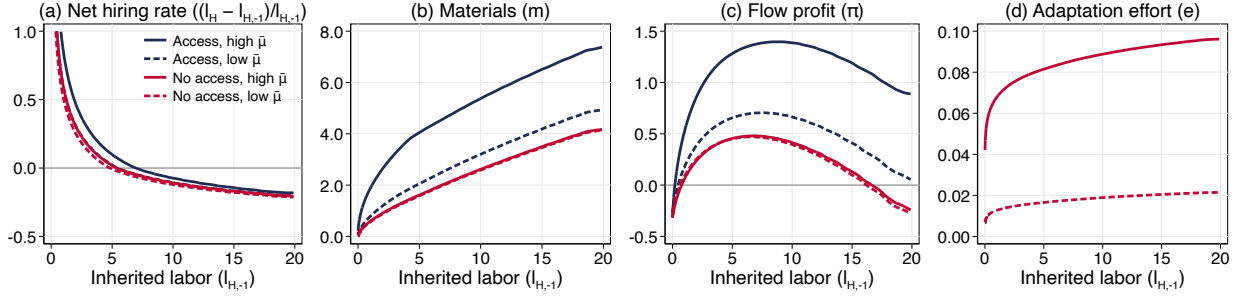


Notes: Solution to the problem of the firm with subcontracting access (for the parameterization discussed in Section 4) plotted against inherited labor $l_{H,-1}$ for two different levels of productivity z , two different types $\bar{\mu}$ (the highest and the lowest) and no transitory shock $\varepsilon = 1$. Panel (a) shows the shadow cost of labor over the market wage $\tilde{w}(z, \varepsilon, l_{H,-1}, \bar{\mu})/w$; Panel (b) shows the net hiring rate $(l_H(z, \varepsilon, l_{H,-1}, \bar{\mu}) - l_{H,-1})/l_{H,-1}$; Panel (c) shows the subcontracted labor $l_S(z, \varepsilon, l_{H,-1}, \bar{\mu})$, and Panel (d) its share in total production labor; Panels (e) and (f) show the delegation cutoff $\hat{\mu}(z, \varepsilon, l_{H,-1}, \bar{\mu})$ and the yield from the marginal delegated task $\phi\tilde{\gamma}(\hat{\mu}(z, \varepsilon, l_{H,-1}, \bar{\mu}); \bar{\mu})$; Panel (g) shows expenditures on materials $m(z, \varepsilon, l_{H,-1}, \bar{\mu})$; and Panel (h) shows flow profits $\pi(z, \varepsilon, l_{H,-1}, \bar{\mu})$.

Policy functions. Figure 8 presents the numerical solution under the calibration discussed in Section 4 for firms with subcontracting access, and different productivity levels and codifiability types. Panel (a) shows the shadow cost of in-house labor relative to the wage, which decreases with inherited labor. Firms that start the period below their desired scale hire, and the hiring friction makes the marginal worker cost more than the wage; firms that start above it reduce labor, and firing costs lower the shadow cost of labor, potentially below the wage. The net hiring rate in Panel (b) tracks these patterns. Higher productivity shifts both curves to the right because the frictionless desired scale is larger.³⁸ In Panel (c), subcontracted labor rises with inherited labor. Inputs are complements in production, so a firm with more workers operates at a larger scale and needs more labor in each task it still delegates, even though it delegates fewer of them. The difference between solid and dashed lines shows that high-type firms delegate more at every state, which captures specialization. In Panel (d), the share of subcontracted labor falls with inherited labor,

³⁸The points where the curves cross their reference lines in Panels (a) and (b) need not coincide, since the shadow cost also includes the expected marginal cost of future in-house labor adjustments.

Figure 9: Decision rules by subcontracting access.



Notes: Solution to the problem of the firm (for the parameterization discussed in Section 4) plotted against inherited labor $l_{H,-1}$ for firms with and without subcontracting access, two different types $\bar{\mu}$ (the highest and the lowest), the high productivity level z of Figure 8 and no transitory shock $\varepsilon = 1$. Panel (a) shows the net hiring rate $(l_H(z, \varepsilon, l_{H,-1}, \bar{\mu}) - l_{H,-1})/l_{H,-1}$; Panel (b) shows expenditures on materials $m(z, \varepsilon, l_{H,-1}, \bar{\mu})$; Panel (c) shows flow profits $\pi(z, \varepsilon, l_{H,-1}, \bar{\mu})$; and Panel (d) shows labor effort $e(z, l_{H,-1}, \bar{\mu})$, which is defined only for firms without access.

which reflects flexibility instead. Small firms looking to expand get closer to their desired scale through subcontracting, while firms with excess labor use their own workers rather than keep them idle. Panels (e) and (f) show that firms short of labor delegate more tasks, up to their whole delegable segment, including some the subcontractor performs worse than they do (marginal yield below one), whereas firms with excess labor retain tasks the subcontractor performs better (marginal yield above one), forgoing some specialization gains. This is the interaction between flexibility and specialization: when the shadow cost of labor is high, delegating low-codifiability tasks is cheaper than hiring; when it is low, producing high-codifiability tasks in-house is cheaper than firing. Materials in Panel (g) are proportional to revenue. They rise with scale, whether it comes from in-house or subcontracted labor, and with productivity, whether from z or specialization gains. Last, profits in Panel (h) are hump-shaped in inherited labor: they rise until decreasing returns to scale and firing costs take over. Productivity is the main driver of profits, and the type gap between solid and dashed lines highlights sizable gains from subcontracting.

Figure 9 compares the policy functions of firms with and without subcontracting access across codifiability types. Panel (a) shows that the net hiring rate is similar across types and access status. In-house adjustment is always present, and firms with access hire slightly more. In Panel (b), the materials of firms without access overlap, since their static choices do not depend on type, while firms with access use more materials for the same reason they hire more: gains from subcontracting increase overall scale. Panel (c) shows that firms with access earn larger profits at every level of inherited labor.³⁹ This gap is the consequence of

³⁹The curves of materials and profits for firms without access do not perfectly overlap because in-house labor choices depend on the continuation value, which includes the option of gaining subcontracting access

both specialization and flexibility gains, which are larger for high-type firms. Last, Panel (d) shows that labor effort tracks these profit differences: effort is larger for high-type firms.

3.3 Closing the Model

Entrants. Entrants hire their initial workforce without adjustment costs.⁴⁰ An entrant chooses labor effort before drawing its productivity from the entrants' distribution $\hat{G}(z, \varepsilon)$, so the free-entry condition is given by

$$\int \max_{e \geq 0} \left\{ -we + p(e) \mathbb{E}_{\hat{G}}[V_S(z, \varepsilon, 0, \bar{\mu})] + (1 - p(e)) \mathbb{E}_{\hat{G}}[V_{NS}(z, \varepsilon, 0, \bar{\mu})] \right\} dF_{\bar{\mu}}(\bar{\mu}) \leq \kappa_E, \quad (16)$$

with equality whenever $E > 0$, where $F_{\bar{\mu}}$ is the distribution of $\bar{\mu}_i$ implied by $F_{\bar{s}}$, E is the mass of entrants in equilibrium and $e_E(\bar{\mu})$ denotes the optimal effort. Firms enter the economy up to the point where the marginal entrant makes no further gains from entering.

Equilibrium. To economize on notation, let $x = (z, \varepsilon, l_{H,-1}, \bar{\mu})$ collect the idiosyncratic state. Together with the access state a , write policies as $l_H(x, a)$, $m(x, a)$, $l_S(x)$, and let $\tilde{\chi}(x, a) = \chi(z, l_H(x, a), \bar{\mu}, a)$ and $\tilde{e}(x) = e(z, l_H(x, 0), \bar{\mu})$ denote the exit and adaptation decisions at the chosen employment. Integrals against λ run over x at the indicated access status.

I focus on a stationary equilibrium consisting of a set of allocations $\{C, N\}$, prices $\{w, \omega\}$, firm policy functions $\{l_H, m, \hat{\mu}, l_S, e, \chi, e_E\}$, value functions $\{V_{NS}, V_S, \mathcal{V}_{NS}, \mathcal{V}_S, \mathcal{J}\}$, a mass of entrants E , and a stationary distribution $\lambda(z, \varepsilon, l_{H,-1}, \bar{\mu}, a)$ over firm states such that:

1. Given prices, firm policy functions solve the problems in (8), (10), (15) and (16), and the exit decision embedded in the continuation value.
2. The subcontracting price satisfies condition (7), implying $\omega = w$ in steady state.
3. The allocations maximize household utility given prices, with labor supply and consumption satisfying equations (6) and (5).
4. The free-entry condition (16) is satisfied.

next period.

⁴⁰In order to ease notation, let $c(l_H, 0) = 0$, so that $V_{NS}(z, \varepsilon, 0, \bar{\mu})$ and $V_S(z, \varepsilon, 0, \bar{\mu})$ give the value of an entrant once its productivity is drawn.

5. The labor market clears:

$$N = L_S + \int l_H(x, a) d\lambda(x, a) + (1 - \delta_X) \int (1 - \tilde{\chi}(x, 0)) \tilde{e}(x) d\lambda(x, 0) + E \int e_E(\bar{\mu}) dF_{\bar{\mu}}(\bar{\mu}),$$

where $L_S = \int l_S(x) d\lambda(x, 1)$.

6. By Walras' law, the goods market clears:

$$\begin{aligned} Y - C - \int m(x, a) d\lambda(x, a) \\ = \int w \frac{\psi_H}{2} \frac{(l_H(x, a) - l_{H,-1})^2}{l_{H,-1}} \mathbf{1}\{l_H(x, a) > l_{H,-1} > 0\} d\lambda(x, a) + \Lambda \kappa_F + E \kappa_E, \end{aligned}$$

where $\Lambda = \int d\lambda(x, a)$ is the mass of active firms.

7. The stationary distribution is consistent with firm policy functions, the entry mass E , and the exit decision:

$$\begin{aligned} \lambda(z', \varepsilon', l', \bar{\mu}', 1) &= (1 - \delta_X) \int_{\bar{\mu} \leq \bar{\mu}'} (1 - \tilde{\chi}(x, 1)) (1 - \delta) \mathbf{1}\{l_H(x, 1) \leq l'\} F(z', \varepsilon' | z) d\lambda(x, 1) \\ &\quad + (1 - \delta_X) \int_{\bar{\mu} \leq \bar{\mu}'} (1 - \tilde{\chi}(x, 0)) p(\tilde{e}(x)) \mathbf{1}\{l_H(x, 0) \leq l'\} F(z', \varepsilon' | z) d\lambda(x, 0) \\ &\quad + E \hat{G}(z', \varepsilon') \int_{\bar{\mu} \leq \bar{\mu}'} p(e_E(\bar{\mu})) dF_{\bar{\mu}}(\bar{\mu}), \\ \lambda(z', \varepsilon', l', \bar{\mu}', 0) &= (1 - \delta_X) \int_{\bar{\mu} \leq \bar{\mu}'} (1 - \tilde{\chi}(x, 1)) \delta \mathbf{1}\{l_H(x, 1) \leq l'\} F(z', \varepsilon' | z) d\lambda(x, 1) \\ &\quad + (1 - \delta_X) \int_{\bar{\mu} \leq \bar{\mu}'} (1 - \tilde{\chi}(x, 0)) (1 - p(\tilde{e}(x))) \mathbf{1}\{l_H(x, 0) \leq l'\} F(z', \varepsilon' | z) d\lambda(x, 0) \\ &\quad + E \hat{G}(z', \varepsilon') \int_{\bar{\mu} \leq \bar{\mu}'} (1 - p(e_E(\bar{\mu}))) dF_{\bar{\mu}}(\bar{\mu}), \end{aligned}$$

where $F(z', \varepsilon' | z)$ is the transition function for firm-level productivity associated with the combined process in (3).

3.4 Aggregate Implications of Subcontracting

Subcontracting affects aggregate efficiency through the two motives identified above. This subsection traces their impact through two aggregate objects: aggregate productivity and a labor wedge.

3.4.1 Productivity

To better understand the aggregate output losses from costly input adjustment, I derive an aggregate productivity expression in the spirit of [Hsieh and Klenow \(2009\)](#).⁴¹ Let M_t and G_t denote aggregate materials and aggregate physical output,

$$M_t = \int_i m_{it} di, \quad G_t = \int_i g_{it} di,$$

and let $\mathcal{L}_t$ denote aggregate production labor, employed by firms or deployed by the subcontractor,

$$\mathcal{L}_t = \int_i (l_{H,it} + l_{S,it}) di.$$

Input efficiency is defined as the aggregate physical output relative to the firm-level technology evaluated at aggregate inputs:

$$\mathcal{A}_t = \frac{G_t}{M_t^{\alpha_m} \left(\frac{\mathcal{L}_t}{1 - \alpha_m} \right)^{1 - \alpha_m}} = \frac{\mathbb{E}_\varsigma [e^{R(\hat{\mu}_{it})} \tilde{w}_{it}^{\alpha_m}]}{(\mathbb{E}_\varsigma [\tilde{w}_{it}])^{\alpha_m} (\mathbb{E}_\varsigma [h_{it}])^{1 - \alpha_m}}, \quad (17)$$

where $\mathbb{E}_\varsigma$ denotes the cross-sectional average with weights $\varsigma_{it} = \tilde{\ell}_{H,it} / \int_i \tilde{\ell}_{H,it} di$, and

$$h_{it} = \frac{(1 - \alpha_m - \hat{\mu}_{it}) + \hat{\mu}_{it} \tilde{w}_{it} / \omega_t}{1 - \alpha_m}$$

measures the firm's total production labor, its own and the subcontractor's, relative to the labor it would employ performing every task in-house.

Let $Y_t = \int_i z_{it} \varepsilon_{it} g_{it}^\nu di$ denote aggregate output. It admits the aggregate production function

$$Y_t \equiv Z_t \Lambda_t^{1 - \nu} \left(M_t^{\alpha_m} \left(\frac{\mathcal{L}_t}{1 - \alpha_m} \right)^{1 - \alpha_m} \right)^\nu, \quad (18)$$

where $\Lambda_t = \int_i di$ is the mass of active firms. Aggregate productivity Z_t can be written as

$$Z_t = P_t \mathcal{A}_t^\nu, \quad P_t \equiv \frac{Y_t}{\Lambda_t^{1 - \nu} G_t^\nu} = \mathbb{E}_t \left[z_{it} \varepsilon_{it} \left(\frac{g_{it}}{\bar{g}_t} \right)^\nu \right],$$

where $\bar{g}_t = G_t / \Lambda_t$ is the average physical output. Aggregate productivity does not depend

⁴¹All derivations are in [Appendix B.9](#).

on aggregate input levels,⁴² but on two objects: size-weighted productivity P_t , which measures how much output the economy obtains from its physical output, given the mass of firms; and input efficiency $\mathcal{A}_t$, which measures how much physical output it obtains from its materials and production labor.

The first term rewards placing output in the most productive firms. Its decomposition resembles [Olley and Pakes \(1996\)](#), adapted to decreasing returns:

$$P_t = \underbrace{\mathbb{E}_t[z_{it}\varepsilon_{it}] \mathbb{E}_t\left[\left(\frac{g_{it}}{\bar{g}_t}\right)^\nu\right]}_{\text{product of means}} + \underbrace{\text{Cov}_t\left(z_{it}\varepsilon_{it}, \left(\frac{g_{it}}{\bar{g}_t}\right)^\nu\right)}_{\text{covariance}}.$$

It rises with the covariance between productivity and size, which input adjustment frictions weaken by keeping firms away from their desired scale. This is the main channel through which flexibility raises aggregate productivity, since subcontracting lets firms adjust their scale without adjusting their workforce. In the product of the means, $\mathbb{E}_t[z_{it}\varepsilon_{it}]$ only changes through selection, since productivity is exogenous, and the average size weight, $\mathbb{E}_t[(g_{it}/\bar{g}_t)^\nu] \leq 1$, captures the cost of unequal firm sizes under decreasing returns.⁴³

The second term captures how shadow costs and delegation gains are distributed across firms. Without subcontracting, $\mathcal{A}_t = \mathbb{E}_\zeta[\tilde{w}_{it}^{\alpha_m}] / (\mathbb{E}_\zeta[\tilde{w}_{it}])^{\alpha_m} \leq 1$, with equality only when shadow costs are equated across firms. Subcontracting affects it through both motives. Delegation gains increase the output obtained from given inputs, and flexibility can compress the dispersion of shadow costs, since firms delegate more when their in-house labor is most expensive.

3.4.2 Aggregate labor wedge

On top of distorting the allocation of inputs, adjustment frictions shift aggregate labor demand relative to the frictionless case. I summarize this effect by computing the wedge between the marginal revenue product of aggregate production labor and the wage. Let the aggregate labor wedge $\tau_{L,t}$ be implicitly defined by

$$\nu(1 - \alpha_m) \frac{Y_t}{\mathcal{L}_t} = \tau_{L,t} w_t.$$

⁴²Under decreasing returns and a fixed amount of inputs, output rises with the number of firms. Including their mass gives the denominator constant returns to scale in the mass of firms, materials and production labor.

⁴³With constant returns, $\nu = 1$, the size weights $g_{it}/\bar{g}_t$ average one, and the expression is the decomposition of [Olley and Pakes \(1996\)](#). Under decreasing returns, dividing by the average size weight also restores that form, $P_t/\mathbb{E}_t[(g_{it}/\bar{g}_t)^\nu] = \mathbb{E}_t[z_{it}\varepsilon_{it}] + \text{Cov}_t(z_{it}\varepsilon_{it}, (g_{it}/\bar{g}_t)^\nu)/\mathbb{E}_t[(g_{it}/\bar{g}_t)^\nu]$.

Integrating the labor optimality conditions in (12) across firms, the wedge is given by

$$\tau_{L,t} = \frac{\bar{w}_t}{w_t} s_{H,t} + \frac{\omega_t}{w_t} (1 - s_{H,t}), \quad (19)$$

where $\bar{w}_t = \int_i \tilde{w}_{it} l_{H,it} / L_{H,t} di$ is the employment-weighted average shadow cost, $L_{H,t} = \int_i l_{H,it} di$ is aggregate in-house labor, and $s_{H,t} = L_{H,t} / \mathcal{L}_t$ is the in-house labor share of production labor.

The labor wedge is a weighted average of the in-house distortion $\bar{w}_t / w_t$ and the subcontracting price-to-wage ratio ω_t / w_t , with weights given by the in-house and subcontracted shares of total production labor.⁴⁴ The same two motives are present in the wedge. First, a larger presence of subcontracting, driven by either motive, lowers $s_{H,t}$, placing more weight on ω_t / w_t .⁴⁵ Second, flexibility compresses $\bar{w}_t / w_t$: firms expand delegation instead of hiring when their labor is scarce, and contract it instead of firing when their labor is trapped, so shadow costs deviate less from the wage.

3.5 Discussion of Assumptions

I conclude this section by discussing some of the model's assumptions.

Sequential task structure. In the spirit of the literature on global production chains (Costinot, Vogel and Wang, 2013; Baldwin and Venables, 2013), I model the technology of firms as a line of sequential tasks that takes material inputs and transforms them with labor. A segment of these tasks is ‘core’ and must be performed in-house. The existence of such a segment is what gives firms a reason to exist: absent it, subcontractors with access to material markets could produce the good themselves. By Lemma 1, this technology is equivalent to a Cobb–Douglas production function in materials and a continuum of tasks with uniform weights, so a horizontally integrated process with independent tasks, as in the task-based models of Acemoglu and Restrepo (2018, 2019), would deliver the same results. However, assuming that production function directly would not lay the groundwork for preventing the subcontractor from producing the goods.

⁴⁴Without adjustment costs, $\tilde{w}_{it} = \omega_t = w_t$, so $\tau_{L,t} = 1$.

⁴⁵In stationary equilibrium, the representative subcontractor perfectly pools idiosyncratic demand every period, so $\omega_t = w_t$ and the subcontracted component contributes no distortion to $\tau_{L,t}$. Under aggregate shocks, the subcontracting price deviates from the wage, eroding the flexibility value of the subcontracting margin.

Specialization yield profile. I assume a profile of subcontracting yields across tasks such that downstream tasks are more codifiable, and thus better performed by the subcontractor, than upstream ones. This choice is motivated by the evidence on the intensive margin of subcontracting in Section 2.3. A model with a flat yield profile (i.e., $\tilde{\gamma}(u; \bar{\mu}) = \tilde{\gamma}$) would either fail to generate an intensive margin like the one observed in the data or generate it for the wrong reason. If $\phi\tilde{\gamma} > 1$, any firm not adjusting in-house labor would delegate all ‘non-core’ tasks, and only firms undergoing layoffs would retain some (up to the point where $\phi\tilde{\gamma} = \omega/\tilde{w}$). The opposite would happen under $\phi\tilde{\gamma} < 1$, with no firm subcontracting except during periods of substantial hiring. In either case, the intensive margin would only be present among firms adjusting labor, subcontracting would need to be more responsive to shocks, and the flexibility channel would be overestimated. Furthermore, differences in codifiability across tasks are well documented in the ‘make-or-buy’ empirical literature (components that are hard to specify to an outside producer are more likely to be made in-house, see [Monteverde and Teece \(1982\)](#), [Masten \(1984\)](#), [Novak and Eppinger \(2001\)](#)). Moreover, as I show in Appendix B.8, the firm’s problem is isomorphic to one in which the subcontractor performs every delegable task with the same efficiency, and coordinating in-house production requires more labor the more tasks the firm performs itself. Specialization gains can then be interpreted as the coordination costs a firm saves by focusing on fewer tasks.

Codifiability types. In the model, firms are heterogeneous in their codifiability type $\bar{\mu}_i$, the size of the segment they can delegate, which is fixed and drawn at entry. This choice is motivated by the between-firm component of subcontracting expenditure shares accounting for almost 70% of their cross-sectional variance, as reported in Section 2.3. Without types, all cross-sectional dispersion would come from labor adjustment, a within-firm component; and it would also be difficult to quantitatively reproduce the rise of subcontracting shares with sales. Thus, under non-negligible type heterogeneity, the largest firms are, on average, those of the highest type. This aligns with the evidence on the role played by ex-ante heterogeneity in determining which firms grow more and become large ([Sterk, Sedláček and Pugsley, 2021](#)).

Subcontracting access. Firms must exert costly effort to adapt their technology to subcontracting, and succeed with a probability that increases in effort. This approach is consistent with the large adaptation costs that [Bajari, Houghton and Tadelis \(2014\)](#) estimate in construction procurement contracts. This randomness in gaining access, together with the exogenous probability δ of losing it, helps the model to match the extensive margin

of subcontracting by size and the transitions in and out of subcontracting documented in Section 2.3. Granting access to all firms would leave those transitions to fluctuations in the shadow cost $\tilde{w}$, which requires a nearly flat yield profile. The common level $\phi\tilde{\gamma}$ would then generate only one of the two: firms with $\phi\tilde{\gamma} > 1$ would only stop, and firms with $\phi\tilde{\gamma} < 1$ would only start. With a per-period fixed cost à la Melitz (2003), participation would be a deterministic function of the firm's state, and nearly all large, high-type firms would subcontract, against the 70% observed in the top sales decile.

The subcontracting sector. I model a representative subcontractor that prices subcontracting services at its marginal cost of labor, and faces the same labor adjustment frictions as firms. Flexibility is thus not assumed but generated by the pooling of idiosyncratic shocks described in Section 3.2, so ω equals the wage in steady state.⁴⁶ Giving the subcontractor market power would not change the calibration, since a constant markup on ω is indistinguishable from a lower level of the yield profile.⁴⁷

4 Quantification

This section details the choice of parameters for the quantitative analysis, evaluates the model's fit to the data, and studies the determinants of firm size in the calibrated economy.

4.1 Calibration

Prior to discussing the choice of parameter values, I introduce the functional forms that remain unspecified. First, I assume the probability of adapting the technology to subcontracting is $p(e) = 1 - \exp(-b_p \cdot e^{a_p})$, where $b_p > 0$ scales the overall level of access efficiency and $a_p \in (0, 1)$ introduces concavity in effort. Both parameters determine returns to effort, and so the extensive margin of subcontracting services. Second, for the yield profile I assume the exponential form $\tilde{\gamma}(u; \bar{\mu}) = e^{-v/v_0}$ for $v = u/\bar{\mu} \in [0, 1]$, which, by Proposition 1, produces the delegation rule $\hat{\mu} = \bar{\mu} \min \{v_0 \max \{\ln \phi + \ln \frac{\tilde{w}}{\omega}, 0\}, 1\}$. The type $\bar{\mu}$ bounds the delegated weight, and $v_0 > 0$ scales the delegated share $\hat{\mu}/\bar{\mu}$. Delegation depends on the sum of two forces: specialization, since $\ln \phi$ sets delegation when the shadow cost of in-house labor equals the price of subcontracting; and flexibility, as $\ln(\tilde{w}/\omega)$

⁴⁶In Section 6.2 I study the model under one-time aggregate shocks. There the subcontractor's demand fluctuates, forcing it to adjust labor, and ω moves with aggregate subcontracting demand.

⁴⁷With a markup m , firms pay $\omega = (1 + m)w$. Since $\phi l_S = \frac{\phi}{1+m}(1 + m)l_S$, they behave as in the model with level $\phi/(1 + m)$ and a price equal to the wage.

shifts it as the shadow cost moves away from that price. Last, the log ratio of the delegable to the core weight is normal, $\ln(\bar{\mu}/(1 - \alpha_m - \bar{\mu})) \sim \mathcal{N}(m_\mu, s_\mu^2)$, discretized into three types of equal mass.

Assigned parameters. A period in the model is one year. Four parameters, $\{\beta, \theta, \iota, \alpha_m\}$, are set externally and given in Panel (a) of Table 2. The discount factor is $\beta = 0.978$, which implies a 2.3% yearly real interest rate, the same rate used to compute capital costs. The labor disutility scale is normalized to $\theta = 1$, and the Frisch elasticity of labor supply is set to $\iota = 0.5$, which is standard in the macroeconomic literature. Neither θ nor ι affects the calibration, because all targeted moments are invariant to the size of the economy. They matter only for aggregate quantities in the counterfactual exercises.⁴⁸ Last, the share of materials in the bundle of variable inputs is fixed to $\alpha_m = 0.363$ to match the aggregate share of materials in labor, materials and subcontracting costs in the data.

Estimated parameters. The remaining 18 parameters, $\Theta = \{\rho, \sigma_\eta, \sigma_\varepsilon, \phi, m_\mu, s_\mu, v_0, b_p, a_p, \delta, \psi_H, \psi_F, \nu, \mu_E, \sigma_E, \delta_X, \kappa_F, \kappa_E\}$, are jointly estimated to target the pervasiveness of subcontracting in the economy, the comovement between inputs and sales, the distribution of sales changes, and the dynamics of entry and exit. All moments are computed from SCIE.

The estimation minimizes the distance between model-generated moments and their empirical counterparts according to the following loss function:

$$\sum_{i=1}^n \left(\frac{\hat{m}_i(\Theta) - m_i}{m_i} \right)^2, \quad (20)$$

where m_i is the empirical moment, $\hat{m}_i(\Theta)$ is its corresponding model-generated moment, and n is the number of targeted moments.

Panel (b) of Table 2 reports the results of the calibration in terms of model fit to the 18 targeted moments and provides the corresponding parameter values. Each parameter is listed next to the moment it is most closely related to, but all are estimated jointly. Block 1 includes the moments associated with the productivity process of firms: the autocorrelation of log sales and the standard deviation of sales growth at one and three years. I include the longer horizon because the relative importance of persistent and transitory shocks in

⁴⁸The free-entry condition pins down the wage without reference to the household, and given the wage, the policies of firms are fully determined. The law of motion of the distribution of firms is then linear in the mass of entrants, so the stationary distribution is that mass times a distribution that does not depend on it. The household's labor supply only determines the mass of entrants, through labor market clearing, and every targeted moment is invariant to it.

Table 2: Parameters and model fit.

<i>Parameter</i>	<i>Description</i>	<i>Value</i>	<i>Target/Source</i>	<i>Data</i>	<i>Model</i>
<i>(a) Externally Calibrated Parameters</i>					
β	Discount factor	0.978	Real interest rate of 2.3%		
θ	Labor disutility scale	1.0	Normalization		
ι	Frisch elasticity of labor supply	0.5	Standard value		
α_m	Materials weight in variable inputs	0.363	Data		
<i>(b) Internally Calibrated Parameters</i>					
<i>Block 1: Productivity process</i>					
ρ	Persistence of z	0.983	Autocorr. log sales, 1 yr	0.95	0.95
σ_ε	Std. dev. of transitory shocks	0.078	Std. dev. sales growth, 1 yr	0.43	0.42
σ_η	Std. dev. of persistent innovations	0.044	Std. dev. sales growth, 3 yr	0.59	0.66
<i>Block 2: Production</i>					
ϕ	Subcontractor's efficiency	1.83	Comovement subcontracting	1.36	1.52
m_μ	Average type	-0.56	Agg. subc. share, subcontractors	0.24	0.27
s_μ	Type dispersion	0.75	Bet.-firm variation subc. cost share	0.69	0.74
v_0	Delegated share scale	1.36	Subc. cost share int. margin, d10-d1	0.12	0.11
b_p	Subc. access scale	0.192	Subcontracting start rate	0.12	0.12
a_p	Subc. access curvature	0.096	Share subcontracting, d10-d1	0.49	0.50
δ	Probability of losing access	0.127	Subcontracting continuation rate	0.85	0.87
ψ_H	Hiring adjustment cost	1.61	Comovement labor, $\Delta \log py > 0$	0.42	0.42
ψ_F	Firing adjustment cost	2.61	Comovement labor, $\Delta \log py < 0$	0.35	0.35
ν	Decreasing returns to scale	0.908	Variable-cost share	0.92	0.88
<i>Block 3: Firm dynamics</i>					
μ_E	Mean of entrants' productivity draw	0.179	Entrants' relative size	0.49	0.50
σ_E	Dispersion of entrants' productivity draw	0.100	Exit rate, entrants rel. to incumbents	1.52	1.51
δ_X	Exogenous exit probability	0.060	Firm exit rate	0.08	0.08
κ_F	Fixed cost	0.165	Exiters' relative size	0.78	0.78
κ_E	Entry cost	6.67	Mean employment	7.77	7.77
<i>Untargeted</i>					
Share of firms subcontracting				0.45	0.41

Notes: The model period is one year. Panel (a) reports externally calibrated parameters. Panel (b) reports internally calibrated parameters, estimated jointly by simulated method of moments as defined by the loss function (20). Comovement values are slope coefficients as in Figure 3, with the labor ones estimated separately for positive and negative sales changes. Relative sizes are mean employment relative to that of incumbents. Cost shares include as costs only the accounts present in the model: labor, materials and subcontracting. The d10-d1 rows compare the top and bottom sales deciles in the share of firms that subcontract and, among firms that subcontract, in the mean subcontracting cost share. For the variable-cost share, sales exclude all other costs in the data, and fixed costs in the model. All aggregate ratios are weighted by the denominator variable.

sales changes varies with the horizon. Together, these moments pin down the persistence ρ and the dispersions σ_η and σ_ε . The model comes close to the autocorrelation (0.95 in both

the model and the data) and to the standard deviation of sales growth at one year (0.42 vs 0.43), and overstates it at three years (0.66 vs 0.59). The persistence and volatility of the productivity process are $\rho = 0.983$ and $\sigma_\eta = 0.044$, while the volatility of transitory shocks is larger: $\sigma_\varepsilon = 0.078$.

Block 2 presents the moments related to subcontracting activity, labor adjustment frictions and returns to scale. The comovement of subcontracting and sales (1.52 vs 1.36) pins down the subcontractor's efficiency ϕ , which defines the elasticity of delegation to the relative cost of in-house production. The between-firm share of the variance in the subcontracting cost share (0.74 vs 0.69) informs the dispersion in types s_μ . The average type m_μ is identified by the aggregate subcontracting share among firms that subcontract (0.27 vs 0.24). The scale of the delegated share v_0 is pinned down by the difference in the intensive margin of subcontracting between the top and the bottom sales deciles (0.11 vs 0.12). To discipline the subcontracting access scale b_p and the probability of losing access δ , I use the empirical probabilities of firms starting to subcontract (0.12 vs 0.12) and of continuing to subcontract across years (0.87 vs 0.85). The curvature a_p governs how strongly access responds to the gains from subcontracting, which rise with firm size, so I target the difference in the share of firms that subcontract between the top and the bottom sales deciles (0.50 vs 0.49).

The comovements of the wage bill with sales in periods of sales expansion (0.42 vs 0.42) and contraction (0.35 vs 0.35) are informative about the responsiveness of labor to shocks, so I use them to discipline the hiring and firing cost scales (ψ_H, ψ_F) . Last, the decreasing returns to scale parameter ν is disciplined by the variable-cost share (0.88 vs 0.92).

The mean and dispersion parameters of the firm types are $m_\mu = -0.56$ and $s_\mu = 0.75$, so the three types are $\bar{\mu} \in \{0.13, 0.23, 0.36\}$. The subcontractor's efficiency, $\phi = 1.83$, exceeds one, so every firm with access delegates a positive weight of tasks, even when the shadow cost of in-house labor equals the price of subcontracting. The scale of the delegated share is $v_0 = 1.36$. The subcontracting access scale and curvature are $b_p = 0.192$ and $a_p = 0.096$. Firms lose access with probability $\delta = 0.127$ each period.⁴⁹ The scales for hiring and firing frictions are $\psi_H = 1.61$ and $\psi_F = 2.61$, so firing a given number of workers costs 62% more than hiring the same number from the same initial workforce. Last, the decreasing returns to scale parameter is $\nu = 0.908$.

Block 3 presents the moments related to firm dynamics. The size of entrants relative to incumbents (0.50 vs 0.49) pins down the mean of entrants' productivity draw μ_E . Its dispersion σ_E determines how many entrants start close to the exit margin, so I target the

⁴⁹Almost every firm with access subcontracts, so the continuation rate of subcontracting is close to $1 - \delta$.

exit rate of entrants relative to that of incumbents (1.51 vs 1.52). The exogenous exit probability δ_X maps into the firm exit rate (0.08 vs 0.08). The fixed cost κ_F shifts the value of the firm and so determines which firms exit voluntarily. Since these are small firms, I use the size of exiting firms relative to incumbents to discipline it (0.78 vs 0.78). The entry cost κ_E defines the equilibrium wage through the free-entry condition, so it is largely pinned down by mean employment (7.77 in both). Entrants draw log persistent productivity with a mean 0.75 long-run standard deviations above the unconditional mean of the process, $\mu_E = 0.179$, and a dispersion of about two fifths of that standard deviation, $\sigma_E = 0.100$. The exogenous exit probability is $\delta_X = 0.060$, and the production fixed cost and the cost of entry are $\kappa_F = 0.165$ and $\kappa_E = 6.67$.

4.2 Untargeted Moments

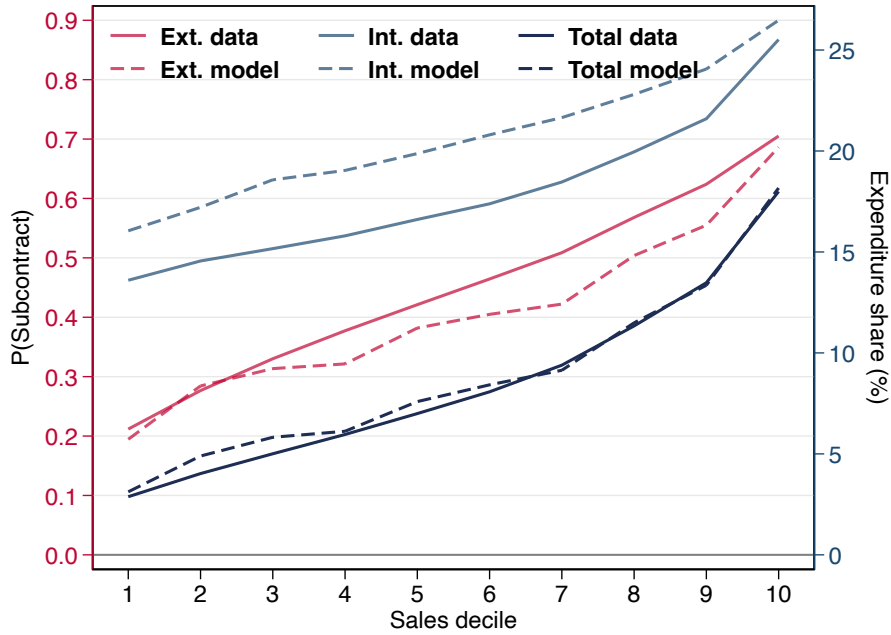
I next compare the calibrated model with the facts from Section 2 that the calibration does not target: the relationship between firm size and subcontracting, the comovement of inputs with sales across horizons, and the relationship between input shares and sales volatility.

Figure 10 compares the model's size-subcontracting relationship to that in the data. The calibration targets the aggregate subcontracting share among firms that subcontract and the differences in both margins between the top and the bottom sales deciles, but not the share of firms that subcontract or how either margin varies across the rest of the size distribution. As in the data, the relationship is increasing and driven by both the extensive and the intensive margins. The model's extensive margin is lower than in the data between the fourth and the ninth deciles, so fewer firms subcontract overall (0.41 vs 0.45). The intensive margin is higher in the model than in the data in every decile, with the smallest gap at the top. The two gaps offset each other, so the calibrated model reproduces the size profile almost exactly.

Figure A.13 in the Appendix compares the comovement of subcontracting and labor with sales across horizons in the model and in the data, as in Figure 4. The calibration only targets the one-year comovements. As in the data, subcontracting comoves more with sales than labor at every horizon, and the gap narrows as the horizon lengthens. The model, however, narrows it more: at ten years, the coefficient for labor is 0.85 in the model against 0.71 in the data, and that for subcontracting is 1.11 against 1.22. Subcontracting thus remains a more important margin of adjustment at longer horizons in the data than in the model.

Table 3 studies the relationship of input usage with observed sales volatility, firm size,

Figure 10: Extensive and intensive margins: data and model.



Notes: In the data, predicted values from regression (1) for the probability of subcontracting, the expenditure share of subcontracting among firms that subcontract, and the product of the two estimates. In the model, the same objects computed by sales decile on a simulated panel at the calibrated parameters. Empirical cost shares have been recomputed by only including as costs the accounts present in the model: labor, materials and subcontracting. ‘Ext.’ stands for extensive margin, ‘Int.’ for intensive margin, and ‘Total’ for the product of the two. All empirical coefficients are significant at the 1% level.

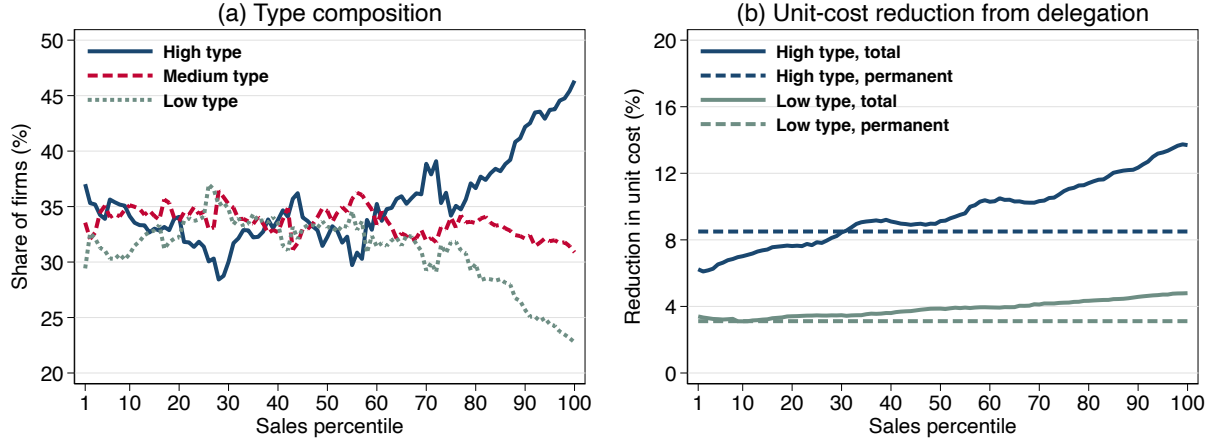
Table 3: Inputs, size, and sales volatility: data and model.

	Subcontracting		Labor		Materials	
	Data	Model	Data	Model	Data	Model
Sales volatility (std.)	1.02	3.67	−1.16	−3.54	0.14	−0.14
Log sales (std.)	4.98	3.22	−9.58	−4.03	4.61	0.81
Volatility × size	0.69	0.45	−0.77	−0.42	0.08	−0.03

Notes: Dependent variables are expenditure shares in p.p. Volatility is measured as in Figure 5. Volatility and log sales are standardized within each sample. Empirical cost shares have been recomputed by only including as costs the accounts present in the model: labor, materials and subcontracting. Data regressions absorb sector × year fixed effects; model regressions absorb year fixed effects only, since the model has no sectors. All coefficients except the interaction term for materials in the data are significant at the 1% level (standard errors clustered at the firm level).

and their interaction. The model reproduces the correlations of subcontracting and labor cost shares with sales and their volatility: larger sales-growth volatility over the previous five years is associated with larger subcontracting expenditure shares and lower labor

Figure 11: Firm types and delegation gains along the size distribution.



Notes: Panel (a) shows the share of firms of each codifiability type $\bar{\mu}$ by sales percentile. Panel (b) shows, for high- and low-type firms with subcontracting access, the average reduction in unit costs from delegation, $1 - e^{-R(\bar{\mu}_{it})}$ (solid lines), and its permanent component, $1 - e^{-R(\bar{\mu}_{\omega})}$ (dashed lines), by sales percentile. Both panels average over 15 adjacent sales percentiles.

expenditure shares, both in the model and in the data. The correlations with volatility are more than three times as large in the model (3.67 vs 1.02 for subcontracting and -3.54 vs -1.16 for labor), so the model overstates the substitution between subcontracting and labor among firms with access. In part, this could reflect that relationships between firms and subcontractors are more complex in reality than in the model: some firm-level fluctuations may not be perfectly absorbed by subcontractors, and subcontracting prices may differ with the size of the task or due to specific contractual arrangements. The coefficients on sales have the same signs as in the data, with smaller magnitudes, especially for labor and materials, and the volatility coefficient for materials is small in both. As in the data, the association of volatility with subcontracting and labor shares is stronger among larger firms, although the interaction terms in the model are about three fifths as large.⁵⁰

4.3 Firm Size and Subcontracting

The calibrated model reproduces how subcontracting rises with firm size (Figure 10). This subsection explores who the large and the small firms are in this economy, and why.

Firm size in the model is driven by productivity and delegation. As in standard models

⁵⁰For a given rate of adjustment, the adjustment costs in the model are proportional to firm size. This specification is common because it makes the speed of adjustment independent of firm size and is consistent with balanced growth. However, adjustment costs that rise more than proportionally with size, such as costs quadratic in the change in labor rather than in its rate, make in-house adjustment relatively costlier for larger firms and can strengthen these interactions.

of firm dynamics, size and productivity are positively correlated, and firms in the top sales decile are about 57% more productive than those in the bottom one. Panel (a) in Figure 11 shows the share of each codifiability type $\bar{\mu}$ by sales percentile. Each type accounts for about a third of firms up to the 60th percentile, while above it the share of high types rises to almost half in the top percentiles. Because types affect a firm only through delegation, the larger share of high types among large firms must reflect that high types gain more from subcontracting than others.

As Proposition 2 shows, delegation lowers the unit cost of firms with access by the factor $e^{-R(\hat{\mu})}$, which makes them larger, and these delegation gains depend on the firm's type. On average, delegation saves 4% of unit costs among low-type firms with access and 11% among high-type ones. At the same shadow cost, a high-type firm delegates 2.8 times as much weight as a low-type firm, and its delegation gains are larger in the same proportion.⁵¹ High-type firms gain more through both motives for subcontracting: from specialization, as the subcontractor outperforms them in more tasks, and from flexibility, as they shift more tasks to it when in-house labor becomes expensive.

Panel (b) in Figure 11 shows how the reduction in unit costs from delegation varies along the size distribution for high- and low-type firms with access. The dashed lines show its permanent component in equation (14), which does not vary with size, so the distance between the solid and the dashed line of each type is the contribution of the state component. The specialization advantage of high-type firms, the distance between the two dashed lines, is therefore the same at every size, while their total advantage widens from about 3 percentage points of unit cost at the bottom of the size distribution to about 9 at the top. This widening comes from flexibility. Large firms are typically hiring, which makes in-house labor expensive for them, and since high-type firms subcontract more weight than low-type firms at the same shadow cost, they avoid more of that expense. Among the smallest firms, instead, high-type firms tend to hold spare labor, which makes in-house labor cheap for them. Subcontracting then saves them less on each task, and they retain some tasks the subcontractor completes more efficiently, so their gains fall below the permanent component. Differences across types in delegation gains are thus largest among large firms, in line with the concentration of high types at the top. Large firms without access also exert the most effort to obtain it.

Small firms are mostly low-productivity firms without access to subcontracting. Without

⁵¹All types share one yield profile, stretched over their delegable segment. By equation (11), the delegation gains in Proposition 2 equal $R(\hat{\mu}) = \int_0^{\hat{\mu}} \ln(\phi\tilde{\gamma}(u;\bar{\mu})\tilde{w}/\omega)du$ at interior and corner solutions alike. Substituting $u = \bar{\mu}v$ gives $R(\hat{\mu}) = \bar{\mu} \int_0^{\hat{\mu}/\bar{\mu}} \ln(\phi\tilde{\gamma}(\bar{\mu}v;\bar{\mu})\tilde{w}/\omega)dv$, where $\tilde{\gamma}(\bar{\mu}v;\bar{\mu})$ depends only on v , and $\hat{\mu}/\bar{\mu}$ only on $\tilde{w}/\omega$.

access, the type does not affect costs, rationalizing the even shares of the three types among them. Subcontracting thus shapes firm size the most where firms can delegate many tasks and in-house labor is expensive, which is at the top of the size distribution.

5 Subcontracting in the Macroeconomy

This section measures the aggregate gains from subcontracting. I begin by closing the subcontracting market in the baseline economy, and document what drives the resulting differences in aggregate productivity. Then, I decompose these gains into flexibility and specialization.

5.1 Aggregate Gains from Subcontracting

I start by computing the counterfactual *no-subcontracting* economy, where subcontracting is unavailable: no firm has access, and all tasks must be produced in-house. Table 4 summarizes the counterfactual and baseline economies in columns 1 and 4, respectively. Subcontracting raises aggregate productivity by 3.9% and output by 9.8%. Output rises by more because materials, production labor and the mass of firms also expand, which reflects general equilibrium forces: higher labor demand raises the wage by 8.3%, and labor supply follows.

The productivity gain comes from input efficiency, which rises by 3.8%, because tasks that the subcontractor performs better are no longer produced in-house. Size-weighted productivity, instead, is unchanged, but it hides two forces that offset each other. The covariance between productivity and size rises from 17.6 to 19.5, as productive firms operate at a larger relative scale when they can expand through delegation. In contrast, the product of the means falls from 82.4 to 80.6, because firm sizes become more unequal.⁵²

Subcontracting also narrows the labor wedge, from 1.10 to 1.08, which brings aggregate labor demand closer to its frictionless level. In the baseline economy, 28% of total production labor is subcontracted, and would have to be employed in-house without subcontracting. This labor carries no distortion, while in-house labor is slightly more distorted. Last, the bottom rows show that without subcontracting, firms become worse at tracking their shocks. The dispersion of sales changes falls from 0.42 to 0.28, while the dispersion of in-house labor changes relative to it rises from 0.40 to 0.55. In the baseline economy, subcontracting changes are more dispersed than sales and in-house labor changes, another sign of its role

⁵²Average productivity, $\mathbb{E}[z\varepsilon]$, rises by 0.2%, while the average size weight, $\mathbb{E}[(g/\bar{g})^\nu]$, falls by 2.4%.

Table 4: Aggregate implications.

	(1)	(2)	(3)	(4)
	No-subc.	Flex-only	Spec-only	Baseline
<i>Aggregates (no-subc. = 100)</i>				
Output, Y	100	101.6	106.8	109.8
Productivity, Z	100	100.5	102.9	103.9
size-weighted productivity, P	100	100.6	99.8	100.1
product of means	82.4	81.6	81.7	80.6
covariance	17.6	19.1	18.1	19.5
input efficiency, $\mathcal{A}^\nu$	100	99.8	103.2	103.8
Mass of firms, $\Lambda^{1-\nu}$	100	100.0	100.4	100.3
Materials, $M^{\nu\alpha_m}$	100	100.5	102.2	103.1
Production labor, $\mathcal{L}^{\nu(1-\alpha_m)}$	100	100.6	101.2	102.1
<i>Labor wedge</i>				
Labor wedge, τ_L	1.101	1.083	1.103	1.077
in-house distortion, $\bar{w}/w$	1.101	1.102	1.103	1.107
in-house share, s_H	1.00	0.81	1.00	0.72
<i>Adjustment</i>				
sd($\Delta \log y$)	0.28	0.41	0.29	0.42
sd($\Delta \log l_H$)/sd($\Delta \log y$)	0.55	0.38	0.56	0.40
sd($\Delta \log \text{subc}$)/sd($\Delta \log y$)	–	2.31	–	1.67
Effective task yield	–	0.99	1.35	1.34

Notes: The top panel reports output, aggregate productivity with its terms (Section 3.4) and the input factors, relative to the no-subcontracting economy. The product of means and the covariance are expressed relative to the size-weighted productivity of the no-subcontracting economy. The entries from other panels are reported in levels: the labor wedge with its components, the dispersion of one-year changes in sales, the dispersion of in-house labor and subcontracting changes relative to it, and the mean effective yield per delegated task. The subcontracting dispersion conditions on positive subcontracting at both ends. *No-subc.* removes access to the subcontracting market. *Flex-only* caps subcontracting yields at one, so that the subcontractor is never more efficient than the firm. *Spec-only* grants the productivity gains of delegation while all labor is in-house.

as an adjustment margin. These results are a first manifestation of the two motives behind subcontracting demand, whose separate contributions I quantify next.

Table 5: Decomposition of subcontracting gains.

	Change	Flexibility (%)	Specialization (%)
Productivity, Z	3.9	18.7	81.3
covariance	1.9	76.9	23.1
product of means	−1.8	54.5	45.5
input efficiency, $\mathcal{A}^v$	3.8	6.7	93.3
Output, Y	9.8	23.3	76.7

Notes: Shapley shares of the change from the no-subcontracting economy to the baseline. The contribution of each component is the average of its gain when added to the *No-subc.* economy and when added to the economy with the other component already present. Changes are in percent, except for the covariance and the product of the means, which are in index points of the no-subcontracting economy's size-weighted productivity.

5.2 Specialization and Flexibility

I next decompose the aggregate subcontracting gains into their flexibility and specialization components. To separate them, I design two counterfactual economies that deviate from the baseline through alternative production technologies. In the *flexibility-only* economy, all yields are capped at one, $\min\{\phi\tilde{\gamma}(u; \bar{\mu}), 1\}$. The subcontractor is exactly as efficient as the firm at the tasks it performs better in the baseline, and remains worse at the rest. In this economy, delegation is valuable only as a way to adjust production scale without altering in-house labor.⁵³ In the *specialization-only* economy, subcontracting access works as a productivity shifter. A firm with access obtains the efficiency gains of the tasks a subcontractor performs better, the term $\exp\{R(\hat{\mu}_\omega)\}$, the permanent component of the delegation gains in (14), but all the labor used in production is in-house and subject to the same frictions. Delegation raises efficiency but does not make adjusting scale any cheaper.⁵⁴ Applying both modifications at once, capping the yields and keeping all labor in-house, leaves a firm with no reason to delegate. Access is then worth nothing and the economy coincides with the no-subcontracting economy. Relative to it, the gains of the flexibility-only economy measure the flexibility motive in isolation, and those of the specialization-only economy measure the specialization motive in isolation, as these two counterfactuals define

⁵³If all yields were equal to one, every task could be delegated at no efficiency cost. Firms would subcontract all their production and adjust it entirely through the subcontractor, and access would be so valuable that every firm would seek it. Adjustment frictions would disappear in practice. Keeping the losses at low-codifiability tasks gives delegation a cost, so that it remains an adjustment margin rather than a replacement of in-house labor.

⁵⁴Under this setup, any firm with access to subcontracting delegates every task whose subcontracted yield exceeds the in-house one.

them. These gains need not add up to the baseline's, so I split the total gain between the two motives with a Shapley decomposition (Shorrocks, 2013). Each economy is solved in general equilibrium under the baseline parameters.⁵⁵

Columns 2 and 3 of Table 4 present the *flexibility-only* and *specialization-only* economies, respectively. Relative to the no-subcontracting economy, flexibility alone raises aggregate productivity by 0.5% and output by 1.6%, and specialization alone by 2.9% and 6.8%, against the 3.9% and 9.8% gains from the baseline. Table 5 expresses these quantities as shares of the total gain. Flexibility accounts for 19% of the productivity gain and 23% of the output gain, and specialization for the rest. The two motives act through different terms: flexibility accounts for three quarters of the rise in the covariance, and specialization for 93% of the rise in input efficiency. Under flexibility alone, firms use delegation to adjust their scale. The labor wedge is lower (1.083 vs 1.103) and the dispersion of sales changes is greater than in the specialization-only economy (0.41 vs 0.29), as firms can track shocks more cheaply than in-house. The gains of this economy concentrate in size-weighted productivity, which rises by 0.6%, as the covariance between productivity and size rises from 17.6 to 19.1. Because the subcontractor is no longer more efficient than the firm at any task, and firms still delegate some at which it is worse, input efficiency falls by 0.2%. Under specialization alone, the amount delegated does not respond to the state of the firm. The gains come instead from the term $\exp\{R(\hat{\mu}_\omega)\}$ defined in equation (14), which raises the productivity of delegating firms, and they concentrate in input efficiency, which rises by 3.2%. Size-weighted productivity, in turn, falls by 0.2%, as delegation gains make firms with access larger and sizes more unequal. The labor wedge stays at the level of the no-subcontracting economy.

Table 6 shows how task delegation relates to the shadow cost of labor in each economy. While the distribution of relative shadow costs is similar across the three, delegation is not. In the *flexibility-only* economy, task delegation is driven solely by the shadow cost of labor: no firm delegates in the bottom quintile, and the delegated weight rises toward the top. However, delegating a lot is costly, as it requires delegating tasks that are more efficiently performed in-house, without enjoying the gains that the most codifiable tasks carry in the baseline economy.⁵⁶ Consequently, the mean yield never exceeds one, and it only falls from one in the second quintile to 0.988 in the top one. The participation rate is also the lowest among the three economies. By contrast, in the *specialization-only* economy, delegated

⁵⁵The specialization-only economy does not need a subcontracting price, as all that matters is access. The flexibility-only economy features $\omega = w$ by the same arguments as in the baseline model.

⁵⁶This is a consequence of the cap on subcontracting yields in this economy, as delegated tasks can never be performed more efficiently by the subcontractor.

Table 6: Delegation across the shadow cost of labor distribution.

$\tilde{w}$	Q1	Q2	Q3	Q4	Q5
<i>Flexibility-only</i>					
Mean $\tilde{w}/w$	0.881	0.984	1.050	1.121	1.305
$\Pr(\hat{\mu} > 0)$	0.000	0.299	0.341	0.297	0.345
$\mathbb{E}[\hat{\mu} \mid \hat{\mu} > 0]$	–	0.149	0.209	0.219	0.237
Effective task yield	–	1.000	0.998	0.991	0.988
<i>Specialization-only</i>					
Mean $\tilde{w}/w$	0.871	0.982	1.056	1.131	1.296
$\Pr(\hat{\mu} > 0)$	0.284	0.378	0.314	0.513	0.518
$\mathbb{E}[\hat{\mu} \mid \hat{\mu} > 0]$	0.192	0.199	0.197	0.204	0.209
Effective task yield	1.351	1.351	1.351	1.351	1.351
<i>Baseline</i>					
Mean $\tilde{w}/w$	0.860	0.972	1.052	1.124	1.325
$\Pr(\hat{\mu} > 0)$	0.423	0.375	0.314	0.427	0.498
$\mathbb{E}[\hat{\mu} \mid \hat{\mu} > 0]$	0.155	0.192	0.225	0.229	0.251
Effective task yield	1.464	1.370	1.318	1.280	1.265

Notes: Q x stands for the x -th quintile of the shadow cost of labor $\tilde{w}$ distribution. Rows report the mean of $\tilde{w}$ relative to the wage, the participation rate, the mean delegated weight among participants, and the mean effective yield per delegated task among participants, $\mathbb{E}[\exp\{\frac{1}{\hat{\mu}} \int_0^{\hat{\mu}} \ln(\phi\tilde{\gamma}(u; \bar{\mu})) du\} \mid \hat{\mu} > 0]$, computed with each economy's own effective yield schedule. *Flexibility-only* caps subcontracting yields at one, so that the subcontractor is never more efficient than the firm; *specialization-only* grants the productivity gains of delegation while all labor is in-house.

weights and effective yields are nearly flat across quintiles. This is expected, as the amount to subcontract in this environment is only a function of the firm's type. Firms with access pay the full cost of adjustments but receive all the gains from tasks better performed by the subcontractor. Participation, instead, rises toward the top, from 0.28 to 0.52.

The baseline economy combines the patterns from both economies. As under flexibility alone, the delegated weight among participants rises with the shadow cost, from 0.16 to 0.25 (from 0.15 to 0.24 under flexibility alone). As under specialization alone, firms delegate in every quintile, and participation reaches 0.50 in the top one, against 0.34 under flexibility alone. The effective yield per task falls from 1.46 to 1.27, as firms with higher shadow costs delegate less codifiable tasks, but it stays well above one throughout. Firms track their shocks about as much as under flexibility alone, with a dispersion of sales changes of 0.42 against 0.41.

6 The Implications of Omitting Subcontracting

This section studies quantitatively the importance of accounting for subcontracting practices when evaluating the aggregate costs of labor adjustment frictions. I first compare the output and productivity losses associated with firing costs in the baseline economy and in an economy without subcontracting calibrated to the same data. Then, I compare how firms respond to idiosyncratic and aggregate shocks in the two economies.

6.1 Firing Costs Revisited

A researcher who wants to measure the aggregate effects of firing costs needs a model of how firms adjust. I study how much this measurement depends on whether the model accounts for subcontracting. I begin by defining two economies without subcontracting. The *recalibrated* economy is the model without subcontracting, calibrated to the targets of Table 2 that do not involve subcontracting; it is the model that a researcher who ignores subcontracting would use. The *no-subcontracting* economy is the baseline economy with the subcontracting market removed, as in Section 5.1.

Table A.7 in the Appendix compares the two calibrations. Without a flexible margin of adjustment, firms track their shocks much less: at the baseline parameters, closing the subcontracting market lowers the dispersion of sales changes from 0.42 to 0.28. The recalibrated economy restores the volatility of sales with larger transitory shocks, and the comovement between labor expenses and sales with lower adjustment costs. Because firms adjust subcontracting mostly to absorb temporary changes (see Appendix B.11), removing subcontracting also makes sales more persistent, and larger transitory shocks undo this as well.⁵⁷ The standard deviation of transitory shocks nearly doubles, while the long-run dispersion of persistent productivity barely changes. Because transitory shocks barely move in-house labor, matching the comovement between labor expenses and sales requires lower adjustment costs: hiring costs are 17% lower than in the baseline, and firing costs 7% lower.

Table 7 compares the steady states of each economy with and without firing costs. Removing them raises output by 5.4% in the baseline and by 7.7% in the recalibrated economy. The gap is smaller for productivity, 1.32% against 1.52%, but the source of the gain differs. In the baseline, the gain comes mostly from selection. Without firing costs, being an active firm is worth more, so free entry requires a 7.5% higher wage. At this

⁵⁷Larger persistent shocks would also raise the volatility of sales and the comovement between labor expenses and sales. However, they would make sales even more persistent.

Table 7: Removing firing costs.

		No subcontracting	
	Baseline	Same param.	Recalibrated
	(1)	(2)	(3)
<i>Aggregates</i>			
Output, Y	5.4	6.5	7.7
Productivity, Z	1.32	2.12	1.52
size-weighted productivity, P	1.5	2.1	1.4
product of means	1.2	2.1	−0.1
covariance	0.3	0.0	1.5
input efficiency, $\mathcal{A}^\nu$	−0.1	0.1	0.1
Mass of firms, $\Lambda^{1-\nu}$	0.1	−0.6	0.9
Materials, $M^{\nu\alpha_m}$	1.8	2.1	2.4
Production labor, $\mathcal{L}^{\nu(1-\alpha_m)}$	2.1	2.7	2.7
Wage, w	7.5	9.8	10.0
<i>Labor wedge</i>			
Labor wedge, τ_L	−5.9	−8.1	−7.3
in-house share, s_H	5.2	−	−
<i>Firm dynamics</i>			
Exit rate	0.7	1.5	0.3

Notes: Each column compares the steady states of the corresponding economy with and without firing costs ($\psi_F = 0$). The top panel reports the percentage changes in output, aggregate productivity with its terms (Section 3.4), the input factors and the wage. The changes in the product of means and the covariance are expressed relative to size-weighted productivity with firing costs, normalized to 100. The other panels report changes in p.p. *Same param.* removes access to the subcontracting market at the baseline parameters. *Recalibrated* is the model without subcontracting, calibrated to the targets that do not involve subcontracting (Table A.7 in the Appendix).

wage, the least productive firms exit, raising the exit rate by 0.7 p.p.,⁵⁸ and increasing the product of means. In the no-subcontracting economy, the effect on exit is stronger and productivity rises by 2.12%. Hence, the presence of subcontracting markets offsets about 40% of the productivity losses from firing costs. In the recalibrated economy, instead, exit barely rises, and the gain comes from the covariance between productivity and size, as firms

⁵⁸Keeping prices fixed, removing firing costs lowers the exit rate in all three economies, from 7.7% to 7.3% in the baseline, from 7.5% to 7.4% in the no-subcontracting economy and from 7.7% to 6.9% in the recalibrated one.

track their shocks better.⁵⁹ A researcher who ignores subcontracting would thus overstate the productivity losses from firing costs by a sixth and attribute them more to how firms track their shocks than to the survival of unproductive firms.

Inputs account for about three quarters of the output gains, and expand more in the recalibrated economy. There, the wage rises by 10.0%, compared with 7.5% in the baseline, and because production labor follows labor supply, labor contributes 2.7 p.p., against 2.1. Removing firing costs also brings labor demand closer to its frictionless level. The labor wedge falls by 7.3 p.p. in the recalibrated economy, and by 5.9 in the baseline, where subcontracted labor carries no distortion and firms shift 5.2 p.p. of production labor back in-house. A larger mass of firms adds another 0.9 p.p., against 0.1, and materials, which are proportional to output, amplify all these gains. Ignoring subcontracting therefore overstates the output losses from firing costs by 41%.

6.2 Firm-Level and Aggregate Responses

Subcontracting allows firms to track productivity shocks better, but the extent to which it does so can depend on the pervasiveness of the shock. Unlike idiosyncratic shocks, aggregate shocks push the economy out of its steady state and affect prices. This subsection studies how useful subcontracting is for adapting production to each type of shock, and how accounting for this margin of adjustment changes the predicted responses to them.

I study this question by comparing how firms respond to the same productivity shock when it hits a single firm and when it hits all firms at once, in the baseline and the recalibrated economies. In particular, I apply an unexpected shock that multiplies productivity by $A_t = \exp(\Delta_A \rho_A^t)$ at date t , where Δ_A is its size and ρ_A its persistence. When the shock hits a single firm, prices remain at their steady-state values; when it hits all firms, the wage and the price of subcontracting follow their equilibrium paths. For the latter, I compute the response of the economy to a one-time shock under perfect foresight (see Appendix B.10 for details).⁶⁰ I consider a 1% shock, $\Delta_A = 0.01$, either persistent with $\rho_A = 0.9$, or transitory,⁶¹ and focus on the firms active at the time of the shock and follow them from

⁵⁹Average productivity, $\mathbb{E}[z\varepsilon]$, rises by 1.1% in the baseline and by 0.6% in the recalibrated economy. The average size weight, $\mathbb{E}[(g/\bar{g})^v]$, rises by 0.4% in the baseline, but falls by 0.7% in the recalibrated economy, offsetting its selection gain.

⁶⁰Along the transition, the mass of entrants responds to the value of entering with finite elasticity, a modification I introduce for this exercise only. A standard free-entry condition like the one given by (16) can generate abnormal entry and price responses along the transition. Appendix B.10 describes the modified environment.

⁶¹Responses are almost proportional to the size of the shock for shocks of similar sizes, and very similar for a negative shock of the same size.

Table 8: Responses to idiosyncratic and aggregate shocks.

	Persistent			Transitory		
	Idio.	Agg.	Ratio	Idio.	Agg.	Ratio
<i>Impact response, r_0</i>						
Baseline	4.36	1.87	2.33	3.23	1.76	1.83
All firms with access	4.94	1.88	2.62	3.81	1.77	2.15
All firms without access	3.35	1.85	1.81	2.11	1.76	1.20
No subc., recalibrated	3.01	1.82	1.65	2.09	1.74	1.20
<i>Cumulative response, $\bar{r}$</i>						
Baseline	80.25	14.03	5.72	5.25	2.18	2.41
All firms with access	81.79	14.09	5.80	6.04	2.19	2.76
All firms without access	77.96	13.93	5.59	3.90	2.17	1.80
No subc., recalibrated	61.51	13.13	4.68	3.78	1.99	1.90

Notes: Relative responses of output to a 1% productivity shock as defined in (21). The top panel reports the response on impact, r_0 , and the bottom panel the cumulative response, $\bar{r} = \sum_{t=0}^{T-1} \beta^t r_t$, where T is the terminal date of the transition. *Idio.* shocks hit a single firm, and prices remain at their steady-state values; *Agg.* shocks hit all firms at once, and prices follow their equilibrium paths. *Ratio* is the idiosyncratic response over the aggregate one. Persistent shocks have $\rho_A = 0.9$, and transitory shocks $\rho_A = 0$. *Baseline* and *No subc., recalibrated* include all firms active at the time of the shock. *All firms with access* and *all firms without access* place every firm in *Baseline* with and without access to subcontracting at that time. *No subc., recalibrated* is the model without subcontracting, calibrated to the targets that do not involve subcontracting (Table A.7 in the Appendix).

then onwards. For a group $\mathcal{G}$ of these firms, I define the relative response of output at date t , with respect to its path in the absence of the shock, as

$$r_t = \frac{1}{\Delta_A} \ln \left(\frac{\int_{i \in \mathcal{G}} y_{it} di}{\int_{i \in \mathcal{G}} y_{it}^0 di} \right), \quad (21)$$

where y_{it}^0 is the output of firm i at date t in the absence of the shock.⁶²

Table 8 reports the response on impact, r_0 , and the cumulative response, $\bar{r} = \sum_{t=0}^{T-1} \beta^t r_t$, where T is the terminal date of the transition. On impact, the relative response of output to a persistent idiosyncratic shock is 4.36 per unit of increased productivity in the baseline, and 3.01 in the recalibrated economy. Under an aggregate shock, instead, the two responses are almost equal, 1.87 and 1.82. A model without subcontracting thus predicts a similar response to aggregate shocks, but a lower response to idiosyncratic ones, and a smaller

⁶²The output of a firm is zero after it exits.

gap between the two: the ratio of the idiosyncratic to the aggregate response is 2.33 in the baseline and 1.65 in the recalibrated economy. The difference between the two economies is larger under transitory shocks, with ratios of 1.83 and 1.20, in line with firms relying more on subcontracting to absorb transitory changes in sales. The cumulative responses show similar patterns.⁶³

To isolate the role of subcontracting,⁶⁴ I compute the responses of the baseline firms when all of them have access at the time of the shock, and when none of them does, keeping all their other states unchanged. Without access, their relative response on impact is 3.35 to a persistent idiosyncratic shock and 1.85 to an aggregate one, close to that of the recalibrated economy, while with access it is 4.94 and 1.88. Under an idiosyncratic shock, access lets firms expand through the subcontractor faster than by hiring in-house. The results in Table A.8 in the Appendix support this point: in the baseline, the relative response of subcontracted labor on impact is 6.01 per unit of increased productivity, against 2.33 for in-house labor. Under an aggregate shock, instead, all labor becomes more expensive, and subcontracted labor responds no more than in-house labor, 0.43 against 0.43. The difference with the idiosyncratic shock comes from prices: the price of subcontracting rises even more than the wage, as the representative subcontractor must adjust its own workforce to serve all its clients at once. Figures A.14 and A.15 in the Appendix show the transitions after a persistent and a transitory aggregate shock of 1%. On impact, the persistent shock raises the wage by 0.87% and the price of subcontracting by 1.44%. These results broadly connect with the literature on adjustment frictions and aggregate fluctuations. In Khan and Thomas (2008), nonconvex capital adjustment costs at the plant level matter little for aggregate investment in general equilibrium, because prices adjust; here, for the same reason, a margin that circumvents labor adjustment frictions matters little for the response of firms to aggregate shocks. Through the lens of the model, subcontracting appears to be a strong tool for adjusting to idiosyncratic shocks, but a weak one for aggregate shocks.

7 Conclusion

In this paper, I study the role of subcontracting in organizing production and quantify its macroeconomic consequences. I document that larger firms devote a higher percentage of their variable costs to subcontracting. In the top sales decile, 70% of firms subcontract, and

⁶³Under transitory shocks, the cumulative response exceeds the impact one, because the shock leaves firms with more labor, which they reduce only gradually.

⁶⁴Part of the differences between the two economies may come from the recalibration, which lowers returns to scale (Table A.7 in the Appendix).

those that do devote 20% of their variable costs to it. I then show that subcontracting is the input that tracks sales most closely, especially at short horizons, which give more weight to transitory changes in sales, and that subcontracting expenditure shares correlate positively with sales volatility. Labor cost shares display the opposite pattern.

Motivated by this evidence, I build a model of firm dynamics, labor adjustment frictions, and task delegation. Firms can exert labor effort to adapt their technology to subcontracting practices, and, if successful, they can pay a representative subcontractor to perform some of the tasks required for production. Firms also differ permanently in how much of their production they can delegate. Delegated tasks do not require in-house labor, which makes subcontracting a more flexible way to scale production. Moreover, the subcontractor is better than the firm itself at completing certain tasks, which generates gains from specialization. The calibrated model implies that subcontracting raises aggregate productivity by 3.9%. Specialization accounts for most of this gain, as tasks that the subcontractor performs better are no longer produced in-house. Flexibility accounts for a fifth of it, as firms track their shocks more closely and their size moves more with their productivity. Accounting for subcontracting also matters for evaluating labor adjustment frictions, as a model without it overstates the productivity and output losses from firing costs. Moreover, through the lens of the model, subcontracting is a strong tool for adjusting to idiosyncratic shocks, but a weak one for aggregate shocks. Common shocks move the demand for subcontracting of all firms in the same direction, and the subcontractor passes the cost of adjusting its workforce on to its clients precisely when firms need flexibility most. These results suggest that the margins firms use to circumvent adjustment frictions can matter for evaluating labor market regulation, and that the flexibility they give firms need not carry over to the aggregate economy.

Several avenues for future research stem from this paper. The lack of information on suppliers of subcontracting services has placed the focus of the model on subcontracting demand and kept the subcontracting sector deliberately simple. Detailed data on subcontracting transactions could open the door to studying the pricing of these practices, contractual arrangements, and sorting patterns between buyers and suppliers of these services. Such data could also allow testing the shock-pooling mechanism that generates the flexibility of subcontracting in the model, and measuring how much of it survives shocks common to the clients of a subcontractor, such as sectoral or regional shocks. Last, the mechanisms described here apply more generally to any context with input adjustment frictions. Studying the value of subcontracting in mitigating capacity constraints on the capital side and its effects on aggregate investment is another promising line of future research.

References

- Abraham, Katharine G and Susan K Taylor**, “Firms’ use of outside contractors: Theory and evidence,” *Journal of Labor Economics*, 1996, 14 (3), 394–424.
- Acemoglu, Daron and Pascual Restrepo**, “The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment,” *American Economic Review*, 2018, 108 (6), 1488–1542.
- **and** —, “Automation and New Tasks: How Technology Displaces and Reinstates Labor,” *Journal of Economic Perspectives*, 2019, 33 (2), 3–30.
- Antràs, Pol**, “Firms, contracts, and trade structure,” *The Quarterly Journal of Economics*, 2003, 118 (4), 1375–1418.
- Antras, Pol and Elhanan Helpman**, “Global sourcing,” *Journal of Political Economy*, 2004, 112 (3), 552–580.
- Arnarson, Björn Thor, Magnus Buus, Andreas Moxnes, Jakob Munch, and Chong Xi-ang**, “Growing Together and Apart: Scale Economies and Labor Specialization in Global Value Chains,” *CESifo Working Paper*, 2026.
- Asker, John, Allan Collard-Wexler, and Jan De Loecker**, “Dynamic inputs and resource (mis) allocation,” *Journal of Political Economy*, 2014, 122 (5), 1013–1063.
- Autor, David H**, “Outsourcing at will: The contribution of unjust dismissal doctrine to the growth of employment outsourcing,” *Journal of Labor Economics*, 2003, 21 (1), 1–42.
- Bajari, Patrick, Stephanie Houghton, and Steven Tadelis**, “Bidding for incomplete contracts: An empirical analysis of adaptation costs,” *American Economic Review*, 2014, 104 (4), 1288–1319.
- Baldwin, Richard and Anthony J Venables**, “Spiders and snakes: Offshoring and agglomeration in the global economy,” *Journal of International Economics*, 2013, 90 (2), 245–254.
- Baqae, David, Ariel Burstein, Cédric Duprez, and Emmanuel Farhi**, “Consumer Surplus from Suppliers: How Big is it and Does it Matter for Growth?,” *Econometrica*, 2025, 93 (6), 2043–2081.

- Bentolila, Samuel and Gilles Saint-Paul**, “The macroeconomic impact of flexible labor contracts, with an application to Spain,” *European Economic Review*, 1992, 36 (5), 1013–1047.
- Bilal, Adrien and Hugo Lhuillier**, “Outsourcing, inequality and aggregate output,” Technical Report, National Bureau of Economic Research 2021.
- Boar, Corina, Denis Gorea, and Virgiliu Midrigan**, “The aggregate costs of uninsurable business risk,” *Working paper*, 2025.
- Boehm, Johannes and Ezra Oberfield**, “Misallocation in the Market for Inputs: Enforcement and the Organization of Production,” *The Quarterly Journal of Economics*, 2020, 135 (4), 2007–2058.
- Bostanci, Gorkem**, “Productivity gains from labor outsourcing: The role of trade secrets,” *Unpublished manuscript*, 2026.
- Caggese, Andrea and Vicente Cuñat**, “Financing constraints and fixed-term employment contracts,” *The Economic Journal*, 2008, 118 (533), 2013–2046.
- Cahuc, Pierre and Fabien Postel-Vinay**, “Temporary jobs, employment protection and labor market performance,” *Labour Economics*, 2002, 9 (1), 63–91.
- , **Olivier Charlot, and Franck Malherbet**, “Explaining the spread of temporary jobs and its impact on labor turnover,” *International Economic Review*, 2016, 57 (2), 533–572.
- Chen, Jiafeng and Jonathan Roth**, “Logs with zeros? Some problems and solutions,” *The Quarterly Journal of Economics*, 2024, 139 (2), 891–936.
- Ciccone, Antonio**, “Input chains and industrialization,” *The Review of Economic Studies*, 2002, 69 (3), 565–587.
- Cooper, Russell W and John C Haltiwanger**, “On the nature of capital adjustment costs,” *The Review of Economic Studies*, 2006, 73 (3), 611–633.
- Costinot, Arnaud, Jonathan Vogel, and Su Wang**, “An elementary theory of global supply chains,” *Review of Economic Studies*, 2013, 80 (1), 109–144.
- Dhyne, Emmanuel, Amil Petrin, Valerie Smeets, and Frederic Warzynski**, “Multiproduct Firms, Import Competition and Productivity,” *Working paper*, 2024.

- Donangelo, Andres, Francois Gourio, Matthias Kehrig, and Miguel Palacios**, “The cross-section of labor leverage and equity returns,” *Journal of Financial Economics*, 2019, 132 (2), 497–518.
- Goldschmidt, Deborah and Johannes F Schmieder**, “The rise of domestic outsourcing and the evolution of the German wage structure,” *The Quarterly Journal of Economics*, 2017, 132 (3), 1165–1217.
- Grossman, Gene M and Elhanan Helpman**, “Integration versus outsourcing in industry equilibrium,” *The Quarterly Journal of Economics*, 2002, 117 (1), 85–120.
- Hopenhayn, Hugo and Richard Rogerson**, “Job turnover and policy evaluation: A general equilibrium analysis,” *Journal of Political Economy*, 1993, 101 (5), 915–938.
- Hsieh, Chang-Tai and Peter J Klenow**, “Misallocation and manufacturing TFP in China and India,” *The Quarterly Journal of Economics*, 2009, 124 (4), 1403–1448.
- Hubmer, Joachim, Mons Chan, Serdar Ozkan, Sergio Salgado, and Guangbin Hong**, “Scalable versus productive technologies,” *The Wharton School research paper*, 2025.
- Huneus, Federico, Kory Kroft, and Kevin Lim**, “Earnings inequality in production networks,” Technical Report, National Bureau of Economic Research 2021.
- Khan, Aubhik and Julia K Thomas**, “Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics,” *Econometrica*, 2008, 76 (2), 395–436.
- Leon, Andrea Atencio De, Claudia Macaluso, and Chen Yeh**, “Job dynamics with staffed labor,” *Federal Reserve Bank Richmond, Mimeo*, 2024.
- Masten, Scott E**, “The organization of production: Evidence from the aerospace industry,” *The Journal of Law and Economics*, 1984, 27 (2), 403–417.
- Melitz, Marc J**, “The impact of trade on intra-industry reallocations and aggregate industry productivity,” *Econometrica*, 2003, 71 (6), 1695–1725.
- Mertens, Matthias and Benjamin Schoefer**, “From labor to intermediates: Firm growth, input substitution, and monopsony,” Technical Report, National Bureau of Economic Research 2024.
- Monteverde, Kirk and David J Teece**, “Supplier switching costs and vertical integration in the automobile industry,” *The Bell Journal of Economics*, 1982, pp. 206–213.

- Novak, Sharon and Steven D Eppinger**, “Sourcing by design: Product complexity and the supply chain,” *Management Science*, 2001, 47 (1), 189–204.
- Olley, G. Steven and Ariel Pakes**, “The Dynamics of Productivity in the Telecommunications Equipment Industry,” *Econometrica*, 1996, 64 (6), 1263–1297.
- Pijoan-Mas, Josep and Pau Roldan-Blanco**, “Dual Labor Markets and the Equilibrium Distribution of Firms,” Working Paper 2530, CEMFI 2025.
- Roys, Nicolas**, “Persistence of shocks and the reallocation of labor,” *Review of Economic Dynamics*, 2016, 22, 109–130.
- Rubbo, Elisa**, “Fragmentation of production and the wage distribution,” Technical Report, Working Paper 2023.
- Shorrocks, Anthony F.**, “Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value,” *Journal of Economic Inequality*, 2013, 11 (1), 99–126.
- Sterk, Vincent, Petr Sedláček, and Benjamin Pugsley**, “The nature of firm growth,” *American Economic Review*, 2021, 111 (2), 547–579.

Production Beyond the Firm

by Juan Segura

Appendix Materials

A Data Appendix

A.1 Variable definition

Some variables recur in Section 2. I define them here and take them as given in the main text:

- *Sales*: income received from goods sold and services provided to other entities. In order to capture sales related to the production process of the firm, I subtract sales from materials bought and later sold without having been transformed.
- *Labor cost*: total personnel expenses, including salaries, employer social security contributions, post-employment benefits, severance payments, workplace accident insurance, and other staff-related expenditures.
- *Capital cost*: the sum over tangible and intangible capital of the stock times the interest rate plus its depreciation rate, computed as described in Appendix A.2.
- *Intermediate input cost*: the sum of the expenditure categories ‘Materials’, ‘Subcontracting’, ‘Specialized services’, ‘Tools’, ‘Energy’, ‘Transportation’, and ‘Other services’.
- *Materials*: expenses for goods intended to be physically incorporated into the final products. This does not include materials bought to be later sold without being transformed.
- *Subcontracting*: work necessary for the production process itself, for which the cooperation of other companies was obtained, subject to formal commitments or simple agreements. The use of ‘work’ in this definition does not reduce to labor, as completion of such tasks can involve capital.
- *Specialized services*: technical services provided by other companies that the company itself cannot provide through its own resources, such as IT services, laboratory analyses, typographical work, studies, and expert opinions.
- *Tools*: short-lived equipment, such as high-depreciation tools and office supplies, whose useful life, under normal conditions of use, does not exceed one year.
- *Energy*: energy and fluids, such as electricity, fuels or water.
- *Transportation*: travel and accommodation of people, and transportation of goods and materials.

- *Other services*: services not covered by the other categories, such as cleaning, advertising, communications, legal, insurance or IP. It also includes rentals.
- *Total variable cost*: the sum of ‘Labor cost’, ‘Capital cost’ and ‘Intermediate input cost’.
- *Profit*: ‘Sales’ minus ‘Total variable cost’.
- *Expenditure share*: computed as any input expenditure divided by ‘Total variable cost’.
- *Sales share*: computed as any input expenditure divided by ‘Sales’.
- *Profit share*: computed as ‘Profit’ divided by ‘Sales’.

A.2 Capital stocks and depreciation

I build two capital stocks for each firm by perpetual inventory: tangible capital, K_{it}^T , and intangible capital, K_{it}^I . Intangible capital excludes goodwill, and neither stock includes investment properties. Each stock starts at its net book value in 2010, or in the firm’s first year if it enters later. It then evolves as

$$K_{it}^x = \max \{0, (1 - \delta_x)K_{i,t-1}^x + A_{it}^x - D_{it}^x\}, \quad x \in \{T, I\},$$

where A_{it}^x and D_{it}^x are the purchases and disposals of capital of type x by firm i in year t , and δ_x is its depreciation rate.

Each capital type mixes asset classes that depreciate at different rates. I assign each class a standard rate (see Table A.1 below) and aggregate the rates with the composition of net investment in SCIE. In a steady state, the stock of class a equals its net investment over its depreciation rate, $K_a = (A_a - D_a)/\delta_a$, where A_a and D_a are purchases and disposals of class a summed over all firms and years. The depreciation rate of tangible capital is then aggregate depreciation over the aggregate stock,

$$\delta_T = (1 - s_0) \frac{\sum_a (A_a - D_a)}{\sum_a (A_a - D_a)/\delta_a},$$

where the sums run over the classes that depreciate. Land and assets under construction are part of the stock but do not depreciate, and the factor $1 - s_0 = 0.773$ accounts for their share. The same formula over the intangible classes, with $s_0 = 0$, gives δ_I . This yields $\delta_T = 0.056$ and $\delta_I = 0.171$.

Capital expenses are the annual cost of using the two stocks: the return forgone on the funds tied up in capital plus depreciation, $k_{it} = (r + \delta_T)K_{it}^T + (r + \delta_I)K_{it}^I$. I set $r = 0.023$, the average real interest rate in Portugal over 2010 to 2022, computed as the 10-year government bond yield (OECD) minus consumer price inflation (World Bank), both taken from FRED.

Table A.1: Depreciation rates by asset class.

Asset class	Rate	Source
<i>Tangible capital</i>		
Land	0	No depreciation
Buildings and other constructions	0.034	EUKLEMS & INTANProd
Basic equipment	0.126	EUKLEMS & INTANProd
Transport equipment	0.185	EUKLEMS & INTANProd
Administrative equipment	0.215	Gouveia and Pereira (2022)
Biological equipment	0.100	Gouveia and Pereira (2022)
Other tangible assets	0.100	Gouveia and Pereira (2022)
Assets under construction	0	No depreciation
<i>Intangible capital</i>		
Development projects	0.230	EUKLEMS & INTANProd
Software	0.315	EUKLEMS & INTANProd
Industrial property	0.230	EUKLEMS & INTANProd
Other intangible assets	0.126	EUKLEMS & INTANProd

Notes: EUKLEMS & INTANProd rates are those of the market economy in Bontadini, Corrado, Haskel, Iommi and Jonasinio (2023, Table A1). The rate for administrative equipment is the average of the EUKLEMS & INTANProd rates for computing and communication equipment, following Gouveia and Pereira (2022, Table 2). Land and assets under construction do not depreciate; their share in the tangible stock is s_0 . Goodwill and investment properties are excluded.

A.3 Sample selection

I start with the raw firm-level dataset *Sistema de Contas Integradas das Empresas* (SCIE) up to year 2022 after dropping single-person businesses (they present no balance sheet data) and duplicated observations. I also remove any firm-year observations prior to 2010 because that is the first year when detailed intermediate input expenditure data became available. I keep the economic activities where firms sell what they produce and where another firm can carry out part of that production on their behalf, so that subcontracting has a clear meaning. This removes wholesale and retail trade (divisions 45 to 47 of CAE Rev. 3), accommodation and food services (55 and 56), real estate activities (68), and rental and leasing activities (77).⁶⁵ There are in total 2,965,649 firm-year observations. I define sales and total variable costs as described in Appendix A.1 and drop all observations with missing or negative sales or costs, and those where the cost of materials and goods sold is below the cost of materials consumed, which would make the cost of goods resold negative. This leaves 2,607,540 observations. In order to focus on the subset of firms with a standard production structure, I keep observations where labor and capital expenditure shares are strictly positive, and that report at least one full-time paid worker. This reduces the sample to 1,987,709 firm-year observations. Then, I trim sales, individual cost accounts, profits, all expenditure and sales shares top and bottom at 0.5%, and changes in logarithms for sales and individual variable costs at 0.1%. This leaves me with 1,758,186

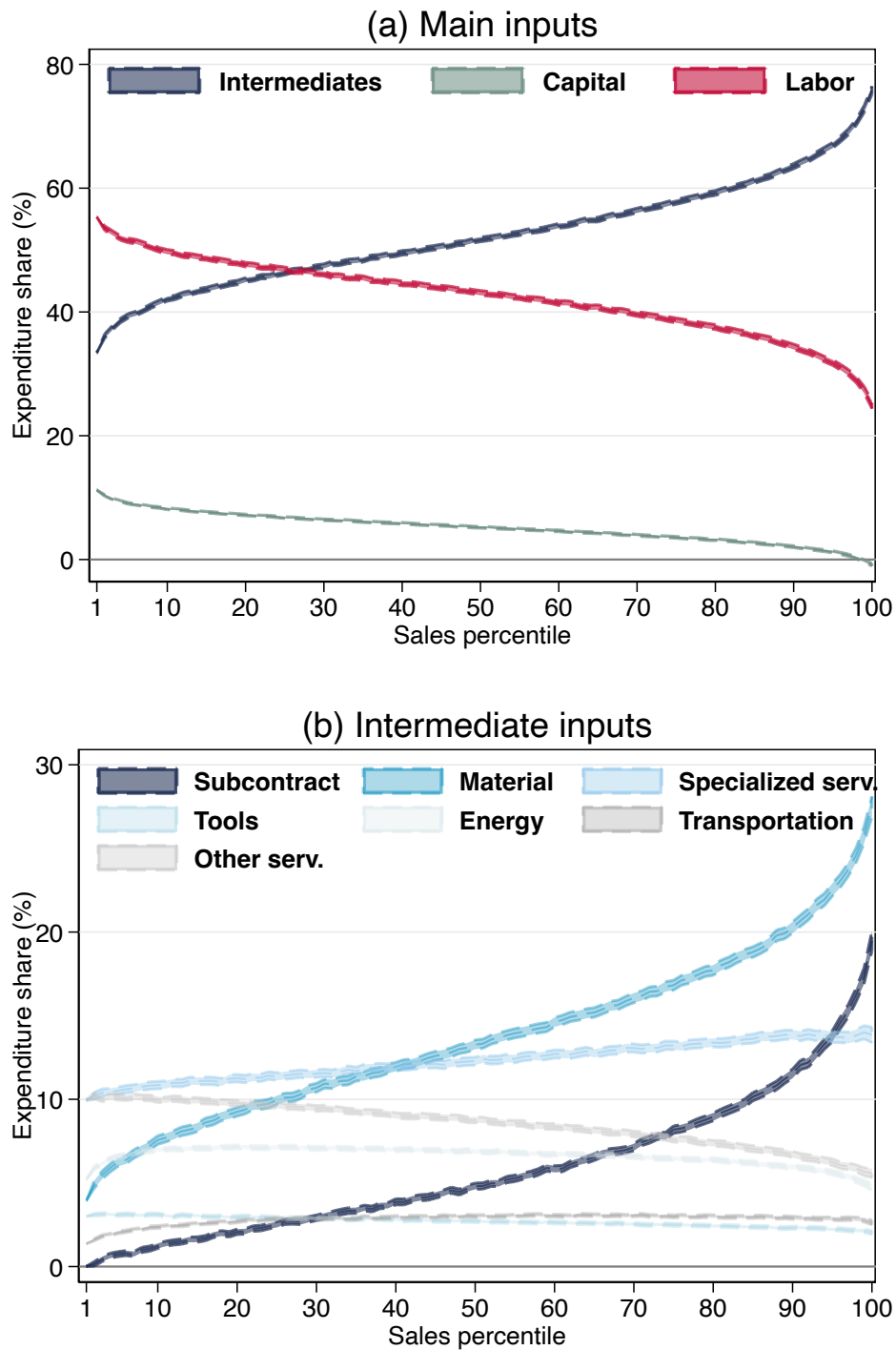
⁶⁵Including these sectors leads to mildly lower use of materials and subcontracting in the sample, in favor of labor, but all empirical results are qualitatively identical.

firm-year observations.

All monetary values are deflated to 2022 euros with the all-items consumer price index for Portugal, annual average, built by chaining the World Bank's annual consumer price inflation.

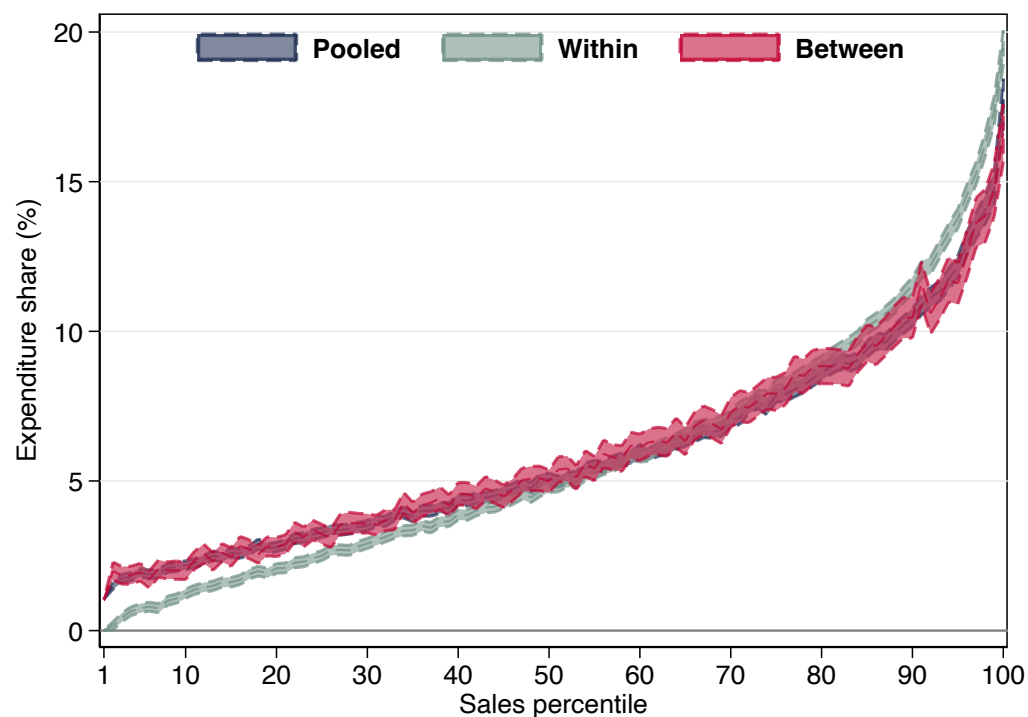
A.4 Additional figures

Figure A.1: Expenditure shares and firm size, within-firm.



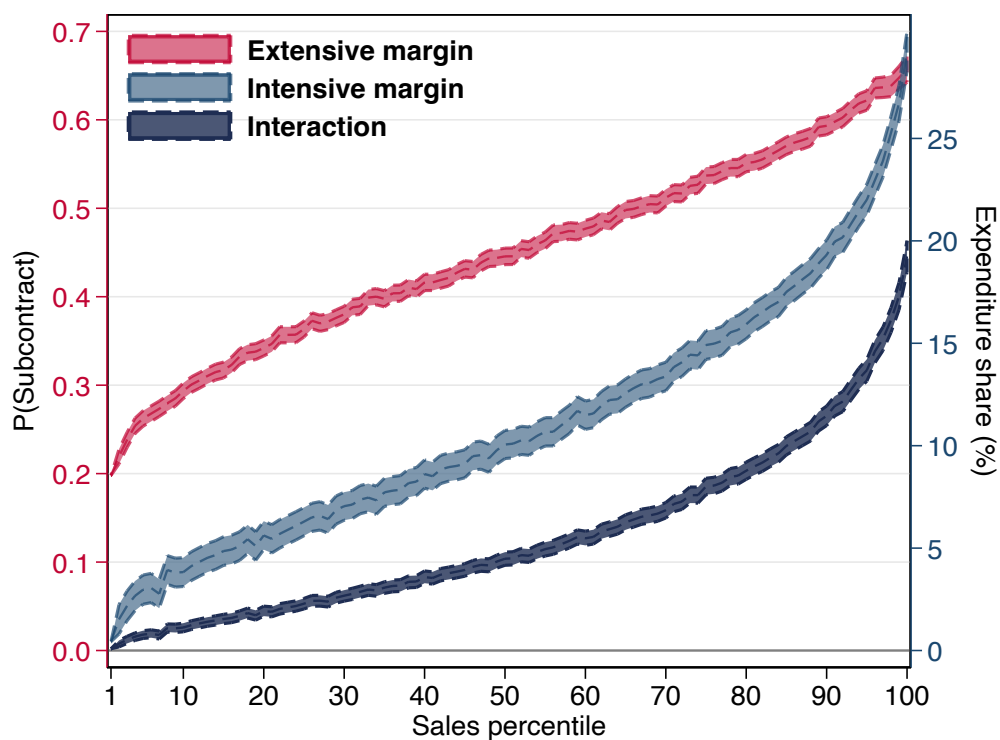
Notes: Predicted values from regression (1) with firm and year fixed effects in place of the sector×year fixed effects. The shaded areas correspond to 95% confidence intervals clustered at the firm. Shares are computed over total variable costs, which are the sum of capital, labor and intermediate input costs. Materials, subcontracting, specialized services, tools, energy, transportation and other services are all the subcategories that comprise intermediate inputs.

Figure A.2: Expenditure share of subcontracting and firm size, within- and between-firm.



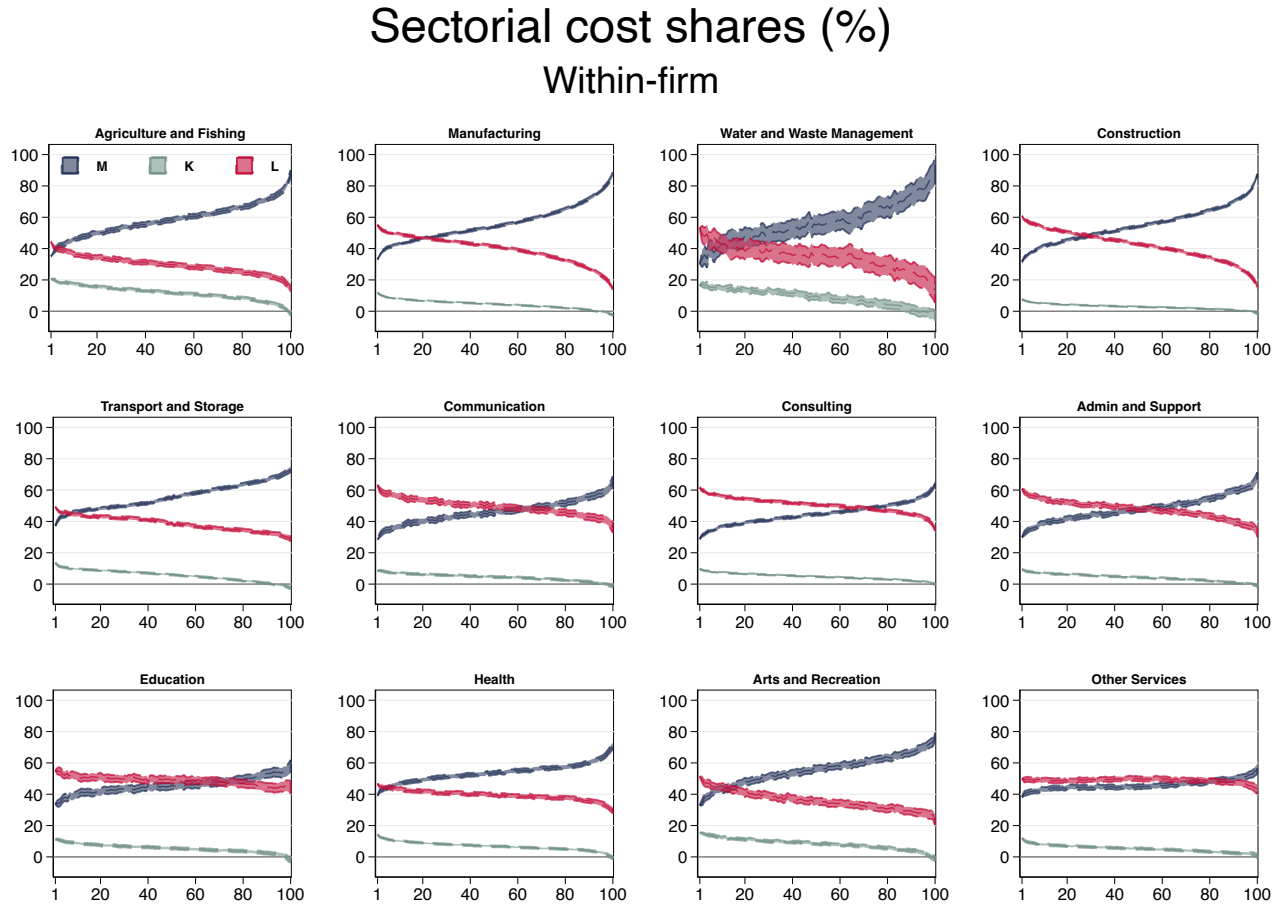
Notes: 'Pooled' shows predicted values from regression (1); 'Within' uses firm and year fixed effects in place of the sector×year fixed effects; 'Between' swaps the sales percentile regressors for the within-firm average sales percentile and keeps the original fixed effects. The shaded areas correspond to 95% confidence intervals clustered at the firm. Shares are computed over total variable costs, which are the sum of capital, labor and intermediate input costs. The sample is limited to firms observed for at least five years.

Figure A.3: Extensive and intensive margins of subcontracting, within-firm.



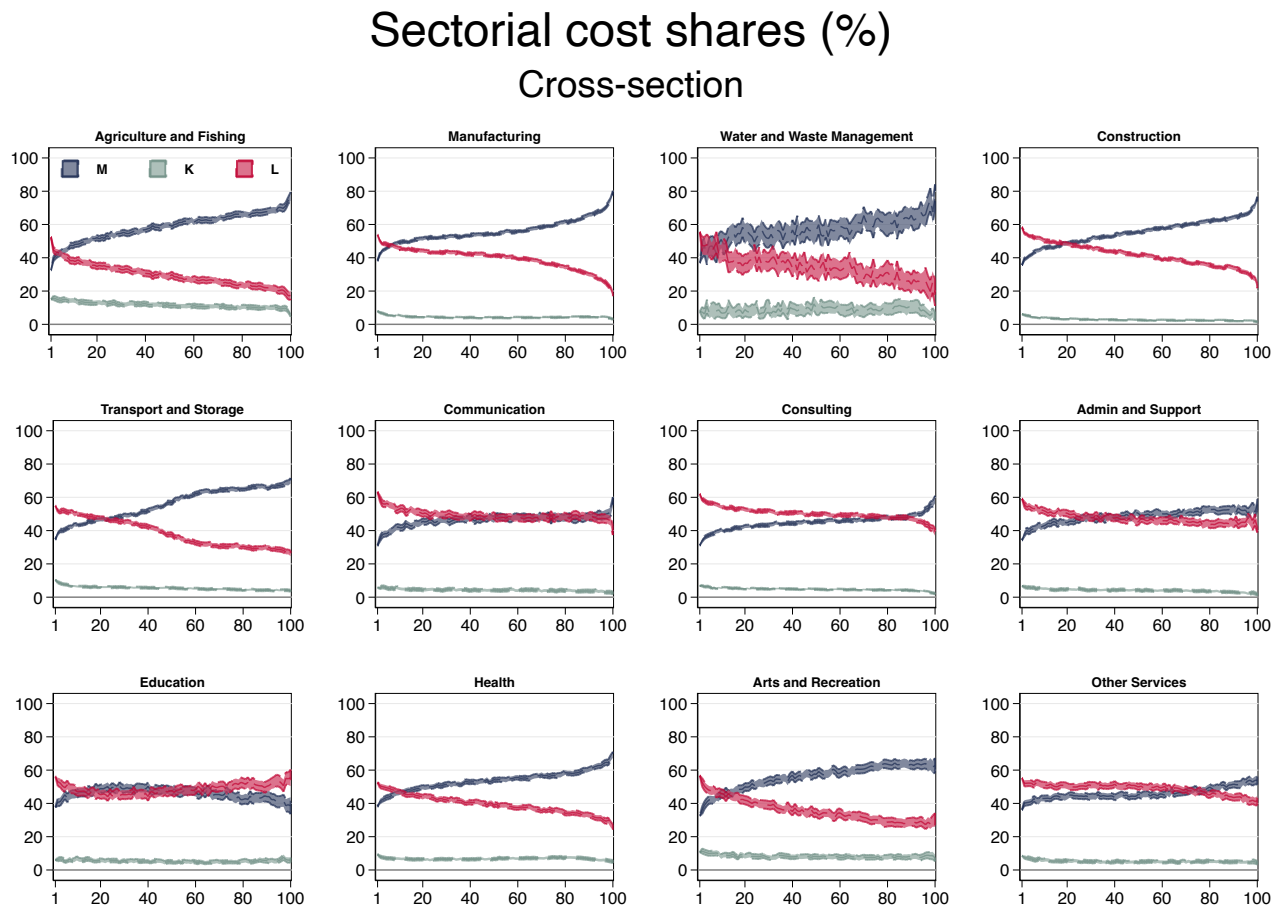
Notes: Predicted values from regression (1) with firm and year fixed effects in place of the sector×year fixed effects, for the empirical probability of subcontracting, the expenditure share of subcontracting, and the product of the two estimates. The shaded areas correspond to 95% confidence intervals clustered at the firm. Confidence intervals for the interaction term are calculated using the Delta method.

Figure A.4: Main expenditure shares and firm size across sectors, within-firm.



Notes: Predicted values from regression (1) with firm and year fixed effects in place of the sector $\times$ year fixed effects, estimated separately for each sector, for different firm inputs. The shaded areas correspond to 95% confidence intervals clustered at the firm. Shares are computed over total variable costs, which are the sum of capital, labor and intermediate input costs. 'M' stands for intermediate inputs, 'K' for capital and 'L' for labor. Mining and energy are left out because they have insufficient observations.

Figure A.5: Main expenditure shares and firm size across sectors, cross-section.

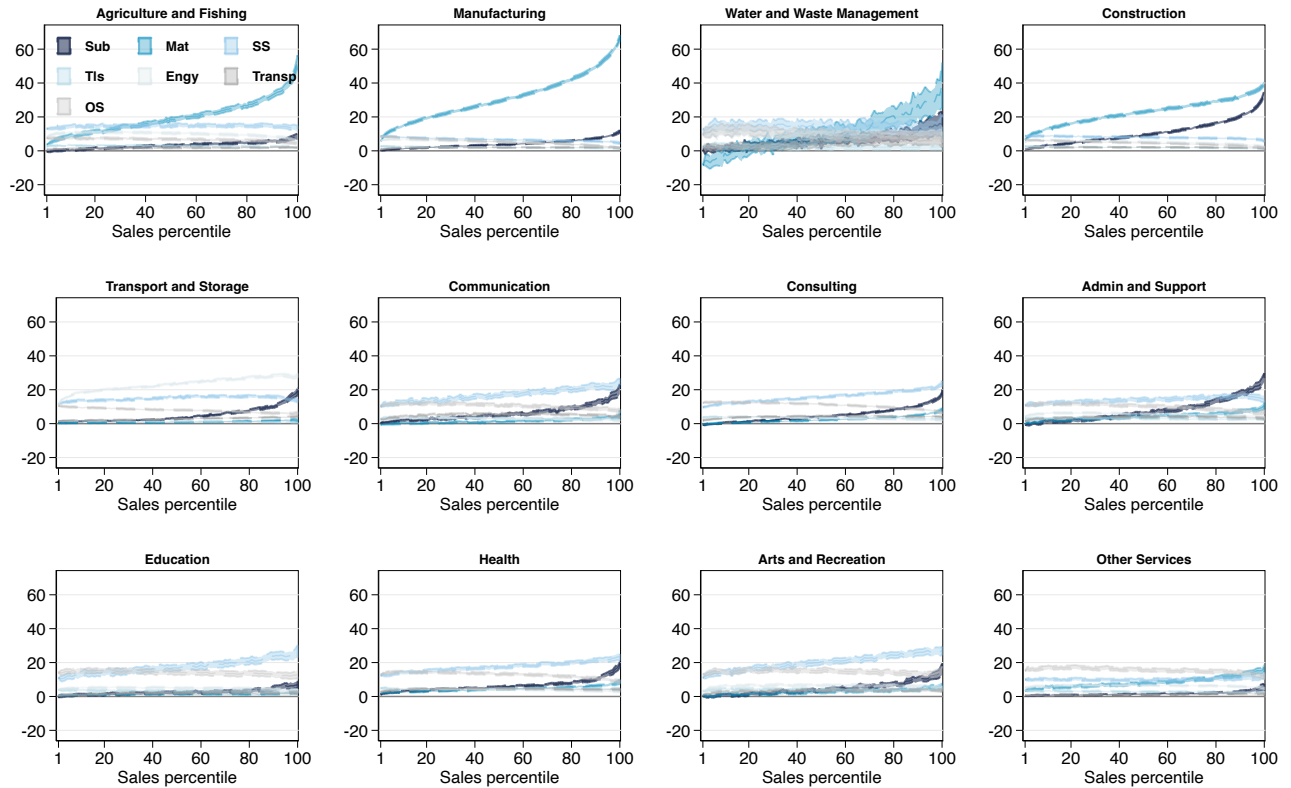


Notes: Predicted values from regression (1), estimated separately for each sector. The shaded areas correspond to 95% confidence intervals clustered at the firm. Shares are computed over total variable costs, which are the sum of capital, labor and intermediate input costs. ‘M’ stands for intermediate inputs, ‘K’ for capital and ‘L’ for labor. Mining and energy are left out because they have insufficient observations.

Figure A.6: Intermediate expenditure shares and firm size across sectors, within-firm.

Sectorial intermediates cost shares (%)

Within-firm

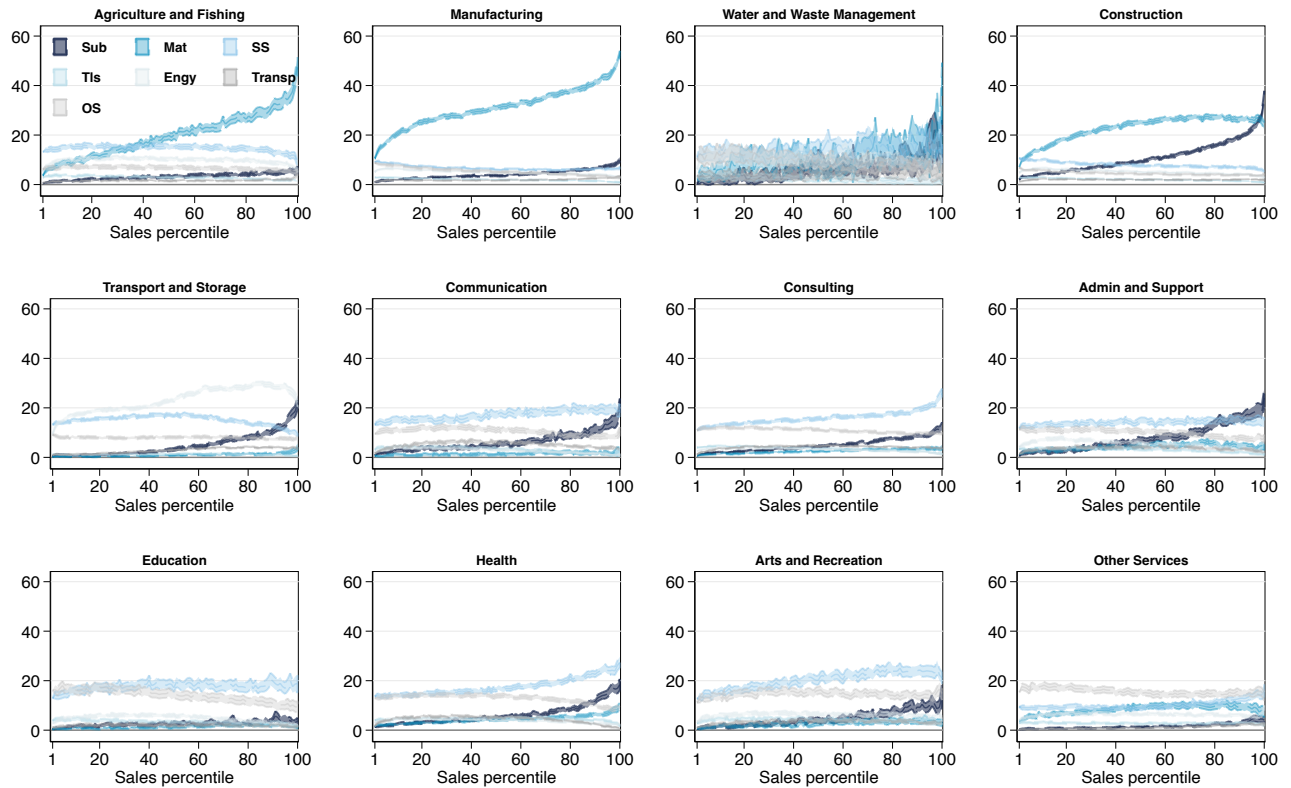


Notes: Predicted values from regression (1) with firm and year fixed effects in place of the sector $\times$ year fixed effects, estimated separately for each sector, for different intermediate firm inputs. The shaded areas correspond to 95% confidence intervals clustered at the firm. Shares are computed over total variable costs, which are the sum of capital, labor and intermediate input costs. Materials, subcontracting, specialized services, tools, energy, transportation and other services are all the subcategories that comprise intermediate inputs. ‘Mat’ stands for materials, ‘Sub’ for subcontracting, ‘SS’ for specialized services, ‘Tls’ for tools, ‘Engy’ for energy, ‘Transp’ for transportation and ‘OS’ for other services. Mining and energy are left out because they have insufficient observations.

Figure A.7: Intermediate expenditure shares and firm size across sectors, cross-section.

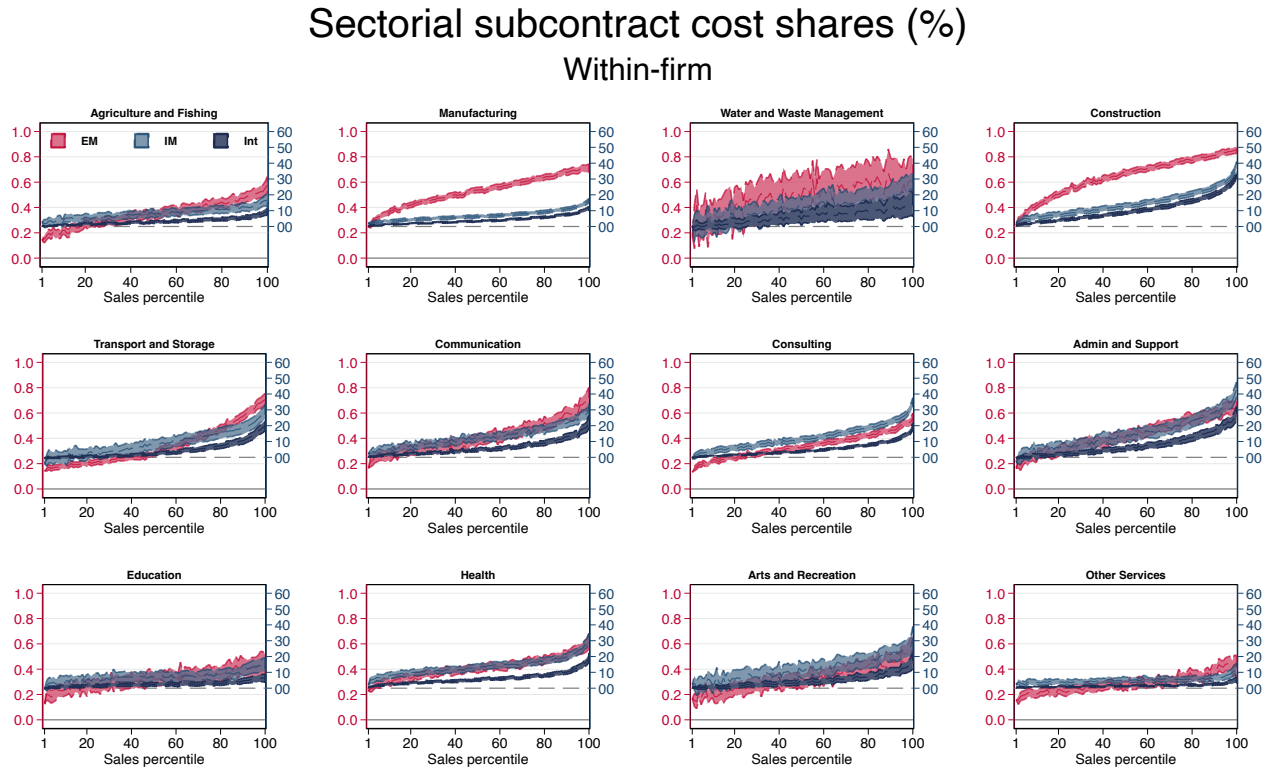
Sectorial intermediates cost shares (%)

Cross-section



Notes: Predicted values from regression (1), estimated separately for each sector, for different intermediate firm inputs. The shaded areas correspond to 95% confidence intervals clustered at the firm. Shares are computed over total variable costs, which are the sum of capital, labor and intermediate input costs. Materials, subcontracting, specialized services, tools, energy, transportation and other services are all the subcategories that comprise intermediate inputs. ‘Mat’ stands for materials, ‘Sub’ for subcontracting, ‘SS’ for specialized services, ‘Tls’ for tools, ‘Engy’ for energy, ‘Transp’ for transportation and ‘OS’ for other services. Mining and energy are left out because they have insufficient observations.

Figure A.8: Extensive and intensive margins of subcontracting across sectors, within-firm.

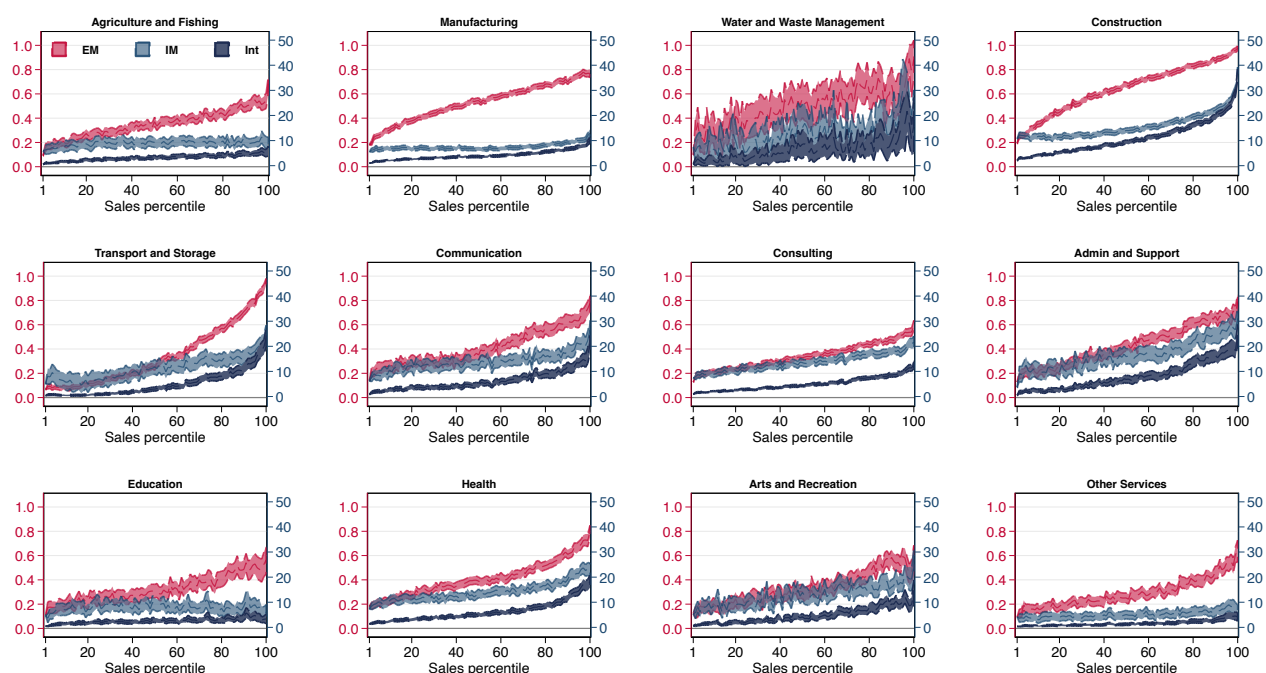


Notes: Predicted values from regression (1) with firm and year fixed effects in place of the sector $\times$ year fixed effects, estimated separately for each sector, for the empirical probability of subcontracting, the expenditure share of subcontracting, and the product of the two estimates. The left y-axis is the empirical probability. The right y-axis is the expenditure share. The shaded areas correspond to 95% confidence intervals clustered at the firm. Confidence intervals for the interaction term are calculated using the Delta method. ‘EM’ stands for extensive margin, ‘IM’ stands for intensive margin and ‘Int’ for interaction. Mining and energy are left out because they have insufficient observations.

Figure A.9: Extensive and intensive margins of subcontracting across sectors, cross-section.

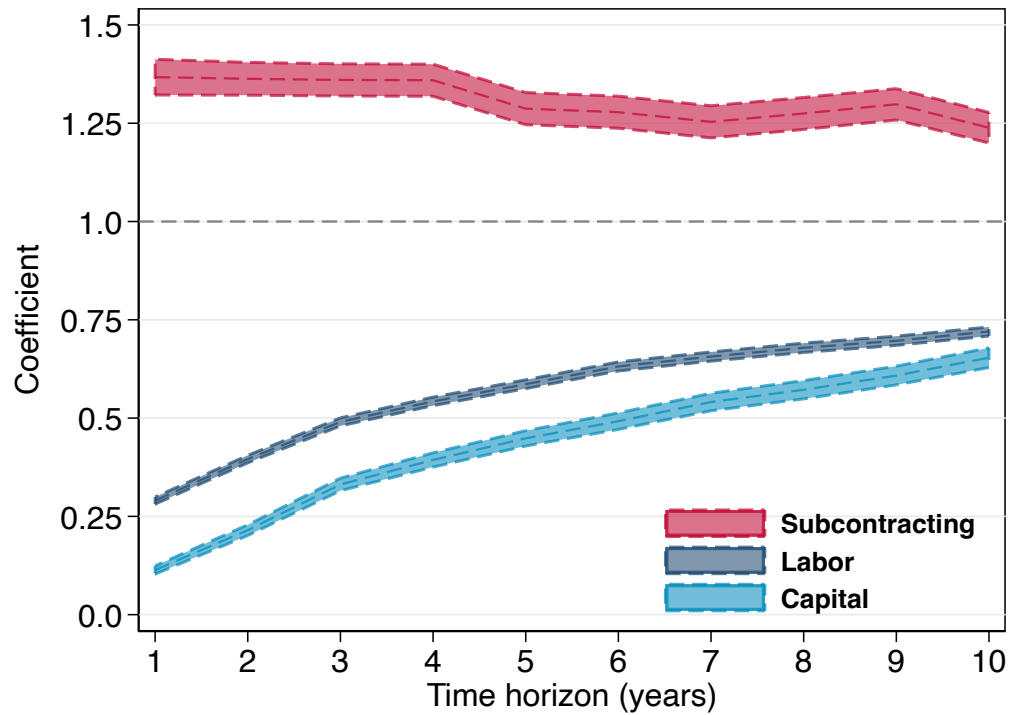
Sectorial subcontract cost shares (%)

Cross-section



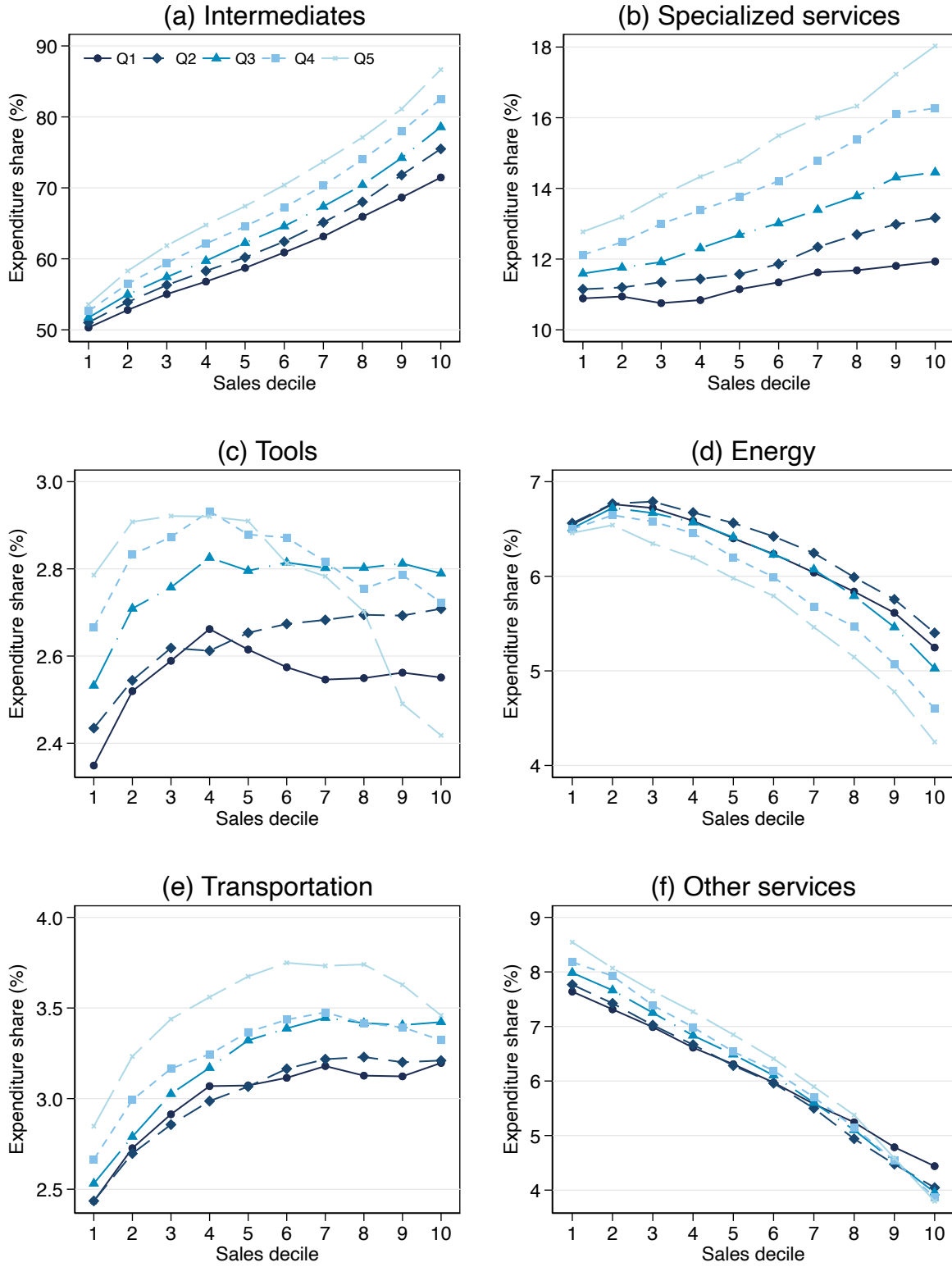
Notes: Predicted values from regression (1), estimated separately for each sector, for the empirical probability of subcontracting, the expenditure share of subcontracting, and the product of the two estimates. The left y-axis is the empirical probability. The right y-axis is the expenditure share. The shaded areas correspond to 95% confidence intervals clustered at the firm. Confidence intervals for the interaction term are calculated using the Delta method. 'EM' stands for extensive margin, 'IM' stands for intensive margin and 'Int' for interaction. Mining and energy are left out because they have insufficient observations.

Figure A.10: Input comovement with sales across horizons, balanced sample.



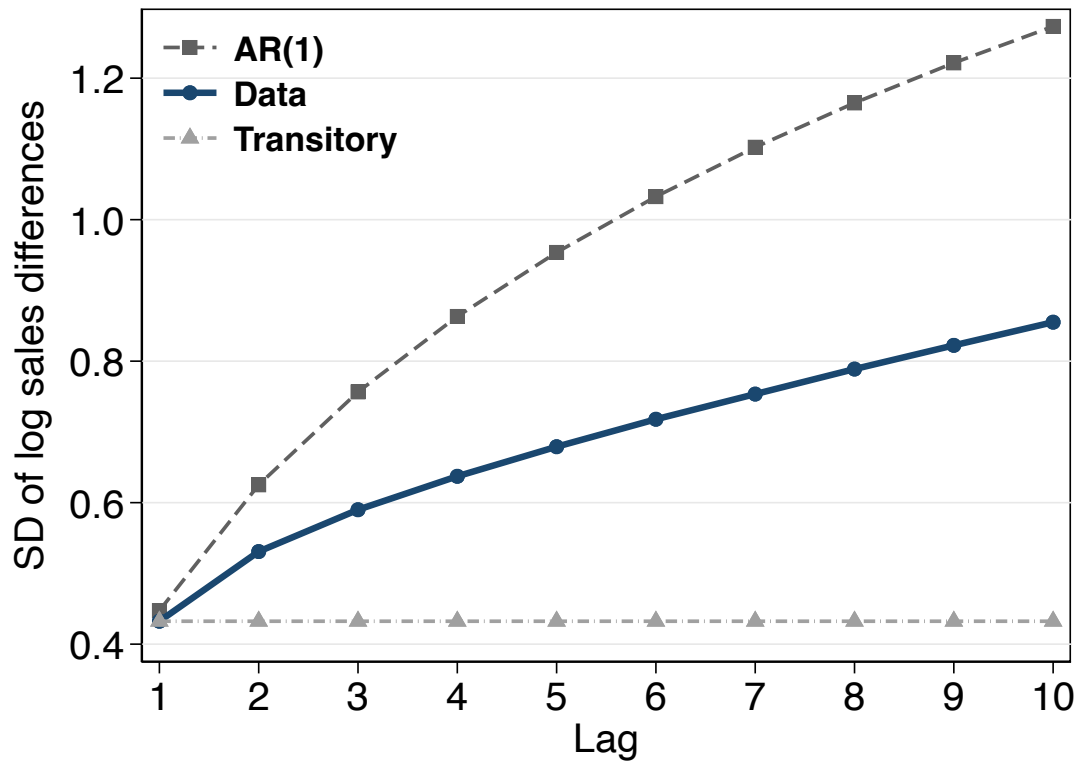
Notes: Estimated coefficients from the regressions in Figure 4, restricted to firm-years observed in each of the ten previous years. Since the data start in 2010, these are the years 2020 to 2022. At the one-year horizon, the sample contains 67,665 observations from 29,817 firms. Shaded areas correspond to 95% confidence intervals clustered at the firm.

Figure A.11: Intermediate inputs and firm volatility.



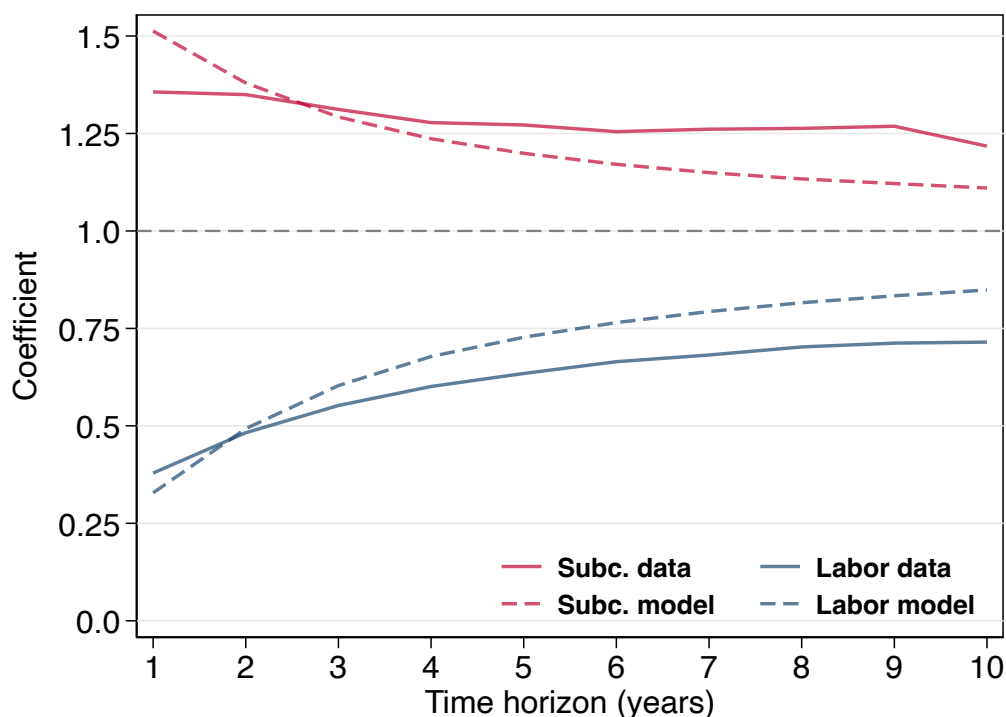
Notes: Estimated as in Figure 5, for total intermediate inputs and the five intermediate accounts other than materials and subcontracting. Q_x in the legend stands for the x -th quintile of the volatility measure distribution.

Figure A.12: Dispersion of sales changes across horizons.



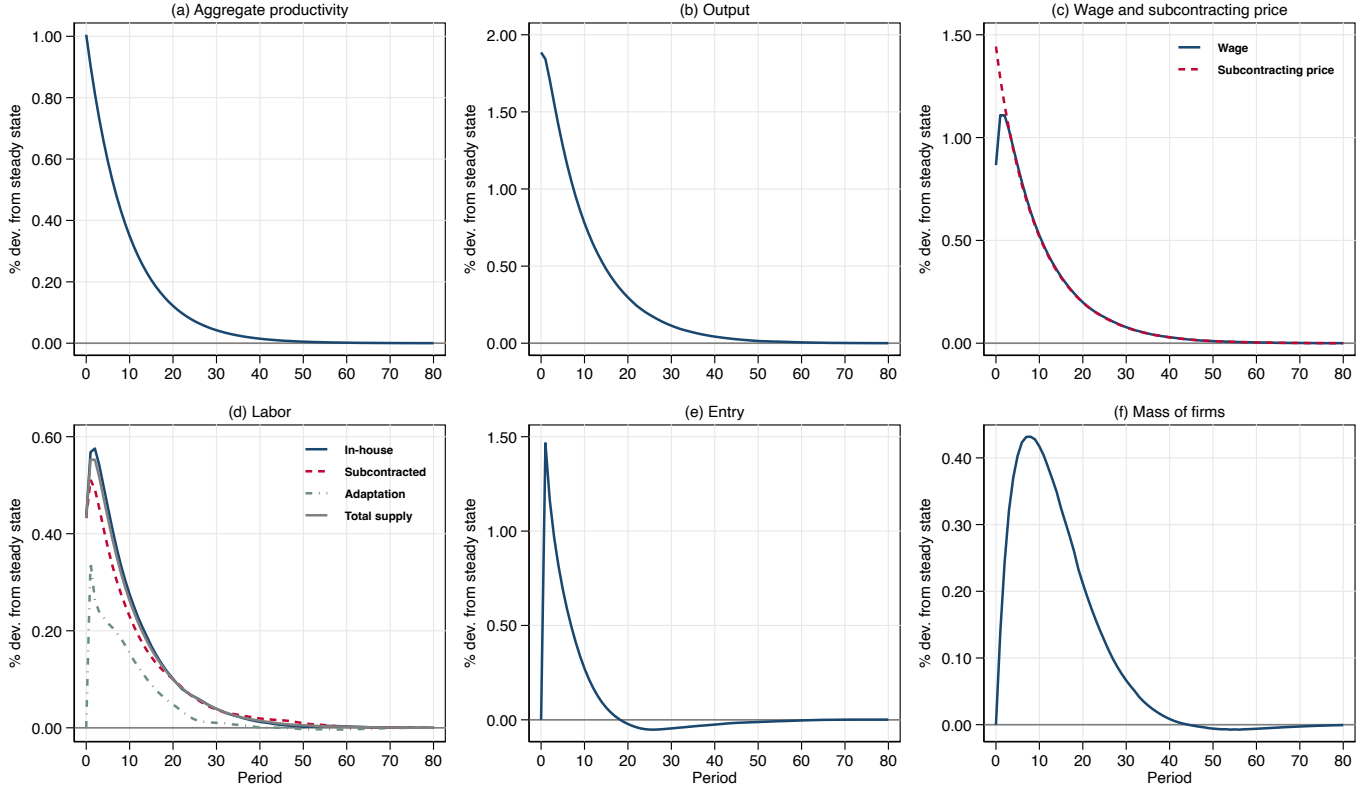
Notes: The figure plots the standard deviation of the k -year difference in log sales, for $k = 1, \dots, 10$, together with the profiles implied by an AR(1) process, with persistence and innovation variance estimated by regressing log sales on their one-year lag, and by a purely transitory process with the variance of the one-year difference. Log sales are net of sector $\times$ year fixed effects.

Figure A.13: Input comovement with sales across horizons: data and model.



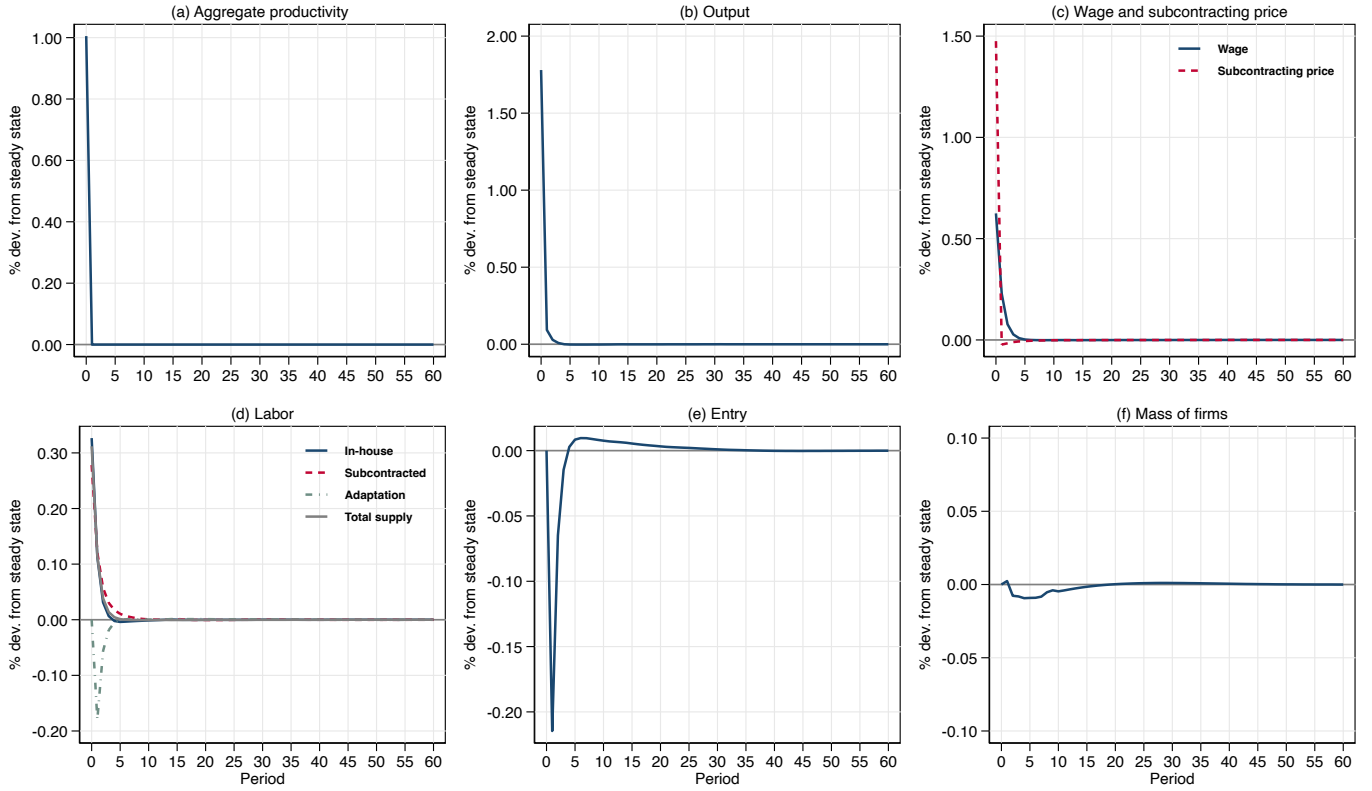
Notes: Coefficients from regressions of k -year log changes in subcontracting expenditures and labor costs on the k -year log change in sales, for $k = 1, \dots, 10$, among firms with positive subcontracting expenditures in both periods, in the data (solid lines) and in the model (dashed lines). The data estimates are those in Figure 4. In both cases, the sample is restricted to log changes in sales below $0.5 \text{sd}_k / \text{sd}_1$ in absolute value, where sd_k is the standard deviation of k -year log changes in sales. Model coefficients are estimated on a simulated panel. The model has no capital, so capital costs are not shown.

Figure A.14: Transitional dynamics, persistent shock.



Notes: The figure plots the transition after an unexpected increase in aggregate productivity of 1% with persistence $\rho_A = 0.9$ for the first 80 periods. All series are percentage deviations from the stationary equilibrium. Panel (a) plots the aggregate productivity shock A_t . Panel (b) plots aggregate output Y . Panel (c) plots the wage w and the subcontracting price ω . Panel (d) plots in-house labor, subcontracted labor, labor spent on adaptation, and total labor supplied by the household. Panel (e) plots the mass of entrants and Panel (f) the mass of active firms. Appendix B.10 describes the environment.

Figure A.15: Transitional dynamics, transitory shock.



Notes: The figure plots the transition after a purely transitory unexpected increase in aggregate productivity of 1% for the first 60 periods. All series are percentage deviations from the stationary equilibrium. Panel (a) plots the aggregate productivity shock A_t . Panel (b) plots aggregate output Y . Panel (c) plots the wage w and the subcontracting price ω . Panel (d) plots in-house labor, subcontracted labor, labor spent on adaptation, and total labor supplied by the household. Panel (e) plots the mass of entrants and Panel (f) the mass of active firms. Appendix B.10 describes the environment.

A.5 Additional tables

Table A.2: Comovement of $\Delta \log py$ and input costs, firms that subcontract.

	$\Delta \log wl$	$\Delta \log k$	$\Delta \log subc$	$\Delta \log mat$
(a) Subcontracting > 0				
$\Delta \log py$	0.411 (0.002)	0.185 (0.002)	1.130 (0.005)	0.956 (0.004)
(b) Subcontracting > 0 , $ \Delta \log py < 0.5$				
$\Delta \log py$	0.379 (0.002)	0.203 (0.003)	1.357 (0.009)	1.035 (0.006)

Notes: Standard errors in parentheses, clustered by firm. Dependent variables: annual log change in labor costs, capital costs, subcontracting expenditures, and material costs, respectively. Explanatory variable: annual log change in sales. Only includes firms with strictly positive subcontracting expenditures in both periods. The last column further conditions on positive material expenditures in both periods ($N = 549,335$ for the first three columns; $N = 337,852$ for materials). Panel (b) limits the sample to observations where the absolute change in log sales is smaller than 0.5 ($N = 462,615$ for the first three columns; $N = 288,234$ for materials). All regressions include sector $\times$ year fixed effects, are run at the firm level, and are unweighted.

Table A.3: Comovement of $\Delta \log py$ and input costs, all firms.

	$\Delta \log wl$	$\Delta \log k$	$\Delta \log subc$	$\Delta \log mat$
(a) All observations				
$\Delta \log py$	0.416 (0.001)	0.170 (0.001)	–	0.938 (0.003)
(b) All observations, $ \Delta \log py < 0.5$				
$\Delta \log py$	0.388 (0.002)	0.197 (0.002)	–	1.018 (0.005)

Notes: Standard errors in parentheses, clustered by firm. Dependent variables: annual log change in labor costs, capital costs, subcontracting expenditures, and material costs, respectively. The column for subcontracting is not estimated because log changes are not well defined across periods with at least one zero. Explanatory variable: annual log change in sales. The last column conditions on positive material expenditures in both periods ($N = 1,430,059$ for the first two columns; $N = 669,810$ for materials). Panel (b) limits the sample to observations where the absolute change in log sales is smaller than 0.5 ($N = 1,186,233$ for the first two columns; $N = 567,074$ for materials). All regressions include sector $\times$ year fixed effects, are run at the firm level, and are unweighted.

Table A.4: Comovement of $\Delta \log py$ and other intermediate inputs, firms that subcontract.

	Specialized services	Tools	Energy	Transportation	Other services
(a) Subcontracting > 0					
$\Delta \log py$	0.567 (0.003)	0.562 (0.004)	0.488 (0.003)	0.687 (0.005)	0.438 (0.003)
Obs.	545,675	540,262	528,346	475,568	546,586
(b) Subcontracting > 0 , $ \Delta \log py < 0.5$					
$\Delta \log py$	0.566 (0.005)	0.603 (0.006)	0.388 (0.004)	0.719 (0.008)	0.376 (0.004)
Obs.	459,832	456,081	447,771	403,381	460,606

Notes: Standard errors in parentheses, clustered by firm. Dependent variables: annual log change in expenditures on specialized services, tools, energy, transportation and other services, respectively. Explanatory variable: annual log change in sales. The sample only includes firms with strictly positive subcontracting expenditures in both periods, and each column further conditions on strictly positive expenditures on the corresponding input in both periods. Panel (b) limits the sample to observations where the absolute change in log sales is smaller than 0.5. All regressions include sector $\times$ year fixed effects, are run at the firm level, and are unweighted.

Table A.5: Comovement of $\Delta \log py$ and other intermediate inputs, all firms.

	Specialized services	Tools	Energy	Transportation	Other services
(a) All observations					
$\Delta \log py$	0.602 (0.002)	0.551 (0.003)	0.518 (0.002)	0.697 (0.003)	0.444 (0.002)
Obs.	1,411,041	1,363,245	1,329,317	1,102,005	1,406,556
(b) All observations, $ \Delta \log py < 0.5$					
$\Delta \log py$	0.610 (0.003)	0.594 (0.005)	0.413 (0.003)	0.733 (0.006)	0.390 (0.003)
Obs.	1,172,473	1,137,714	1,114,475	923,566	1,169,469

Notes: Standard errors in parentheses, clustered by firm. Dependent variables: annual log change in expenditures on specialized services, tools, energy, transportation and other services, respectively. Explanatory variable: annual log change in sales. Each column conditions on strictly positive expenditures on the corresponding input in both periods. Panel (b) limits the sample to observations where the absolute change in log sales is smaller than 0.5. All regressions include sector $\times$ year fixed effects, are run at the firm level, and are unweighted.

Table A.6: Comovement of input expenditures with sales across horizons.

	Horizon (years)									
	1	2	3	4	5	6	7	8	9	10
Labor	0.379 (0.002)	0.482 (0.003)	0.552 (0.003)	0.601 (0.003)	0.634 (0.004)	0.664 (0.004)	0.682 (0.004)	0.702 (0.005)	0.712 (0.005)	0.715 (0.006)
Capital	0.203 (0.003)	0.320 (0.004)	0.411 (0.005)	0.471 (0.006)	0.524 (0.006)	0.552 (0.008)	0.584 (0.009)	0.627 (0.010)	0.641 (0.011)	0.668 (0.012)
Intermediates	0.920 (0.002)	0.958 (0.002)	0.972 (0.002)	0.977 (0.003)	0.983 (0.003)	0.983 (0.003)	0.986 (0.003)	0.986 (0.003)	0.994 (0.004)	0.987 (0.004)
Subcontracting	1.357 (0.009)	1.349 (0.009)	1.311 (0.010)	1.277 (0.011)	1.272 (0.012)	1.254 (0.013)	1.260 (0.014)	1.263 (0.015)	1.268 (0.017)	1.218 (0.019)
Materials	1.035 (0.006)	1.045 (0.006)	1.056 (0.006)	1.054 (0.007)	1.064 (0.007)	1.072 (0.008)	1.071 (0.009)	1.078 (0.009)	1.099 (0.010)	1.081 (0.011)
Specialized services	0.566 (0.005)	0.639 (0.005)	0.680 (0.005)	0.701 (0.005)	0.711 (0.005)	0.719 (0.006)	0.735 (0.006)	0.739 (0.007)	0.752 (0.008)	0.751 (0.009)
Tools	0.603 (0.006)	0.713 (0.006)	0.759 (0.006)	0.789 (0.007)	0.805 (0.007)	0.813 (0.008)	0.819 (0.009)	0.824 (0.010)	0.831 (0.011)	0.816 (0.012)
Energy	0.388 (0.004)	0.478 (0.004)	0.532 (0.005)	0.564 (0.005)	0.579 (0.005)	0.597 (0.006)	0.607 (0.006)	0.622 (0.007)	0.626 (0.008)	0.649 (0.009)
Transportation	0.719 (0.008)	0.828 (0.009)	0.896 (0.009)	0.936 (0.010)	0.951 (0.012)	0.970 (0.013)	0.976 (0.014)	0.979 (0.015)	0.973 (0.017)	0.989 (0.019)
Other services	0.376 (0.004)	0.475 (0.004)	0.542 (0.005)	0.582 (0.005)	0.601 (0.006)	0.628 (0.006)	0.647 (0.007)	0.658 (0.007)	0.668 (0.008)	0.673 (0.009)

Notes: Estimated coefficients from regressions of k -year log changes in expenditures on each input on the k -year log change in sales, with sector $\times$ year fixed effects. Standard errors in parentheses, clustered by firm. The sample includes firms with positive subcontracting expenditures at both ends of the horizon and excludes log changes in sales larger than $0.5 \text{sd}_k / \text{sd}_1$ in absolute value, as in Figure 4. For materials and the five other intermediate accounts, it further conditions on positive expenditures on that input at both ends. The rows for subcontracting, labor and capital are the coefficients plotted in Figure 4.

Table A.7: The recalibrated economy: parameters and fit.

Parameter	Description	Value			Data	Model	
		Baseline	Recalibrated	Target		Baseline	Recalibrated
Block 1: Productivity process							
ρ	Persistence of z	0.983	0.973	Autocorr. log sales, 1 yr	0.952	0.948	0.926
σ_ε	Std. dev. of transitory shocks	0.078	0.144	Std. dev. sales growth, 1 yr	0.432	0.423	0.423
σ_η	Std. dev. of persistent innovations	0.044	0.055	Std. dev. sales growth, 3 yr	0.590	0.661	0.603
Block 2: Production							
ψ_H	Hiring adjustment cost	1.61	1.33	Comovement labor, $\Delta \log \text{py} > 0$	0.415	0.421	0.414
ψ_F	Firing adjustment cost	2.61	2.43	Comovement labor, $\Delta \log \text{py} < 0$	0.353	0.350	0.350
ν	Decreasing returns to scale	0.908	0.872	Variable-cost share	0.917	0.880	0.854
Block 3: Firm dynamics							
μ_E	Mean of entrants' productivity draw	0.179	−0.023	Entrants' relative size	0.485	0.502	0.487
σ_E	Dispersion of entrants' productivity draw	0.100	0.123	Exit rate, entrants rel. to incumbents	1.521	1.514	1.521
δ_X	Exogenous exit probability	0.060	0.060	Firm exit rate	0.078	0.077	0.077
κ_F	Fixed cost	0.165	0.211	Exiters' relative size	0.776	0.776	0.773
κ_E	Entry cost	6.67	3.92	Mean employment	7.77	7.77	7.77

Notes: The recalibrated economy is the model without subcontracting, calibrated to the targets of Table 2 that do not involve subcontracting. The baseline columns repeat the corresponding entries of that table, and the external parameters are the same.

Table A.8: Responses of production labor to idiosyncratic and aggregate shocks.

	Persistent			Transitory		
	Idio.	Agg.	Ratio	Idio.	Agg.	Ratio
<i>Impact response, r_0</i>						
Baseline	3.37	0.43	7.82	2.08	0.31	6.67
All firms with access	4.09	0.45	9.10	2.81	0.32	8.72
All firms without access	2.12	0.41	5.15	0.71	0.31	2.27
No subc., recalibrated	1.87	0.42	4.41	0.74	0.32	2.28
Baseline, in-house labor	2.33	0.43	5.38	0.86	0.33	2.64
Baseline, subcontracted labor	6.01	0.43	13.92	5.14	0.28	18.51
<i>Cumulative response, $\bar{r}$</i>						
Baseline	78.00	1.80	43.28	4.40	0.80	5.49
All firms with access	79.92	1.88	42.54	5.37	0.82	6.56
All firms without access	75.16	1.69	44.46	2.77	0.79	3.50
No subc., recalibrated	58.75	1.01	58.43	2.79	0.64	4.37
Baseline, in-house labor	75.28	2.02	37.25	3.53	0.83	4.26
Baseline, subcontracted labor	84.77	1.30	64.99	6.61	0.73	9.00

Notes: Relative responses of production labor to a 1% productivity shock, defined as in (21) with production labor in place of output. Production labor is the in-house labor of firms plus the labor the subcontractor uses for their orders. The last two rows of each panel split the response of the baseline's production labor into in-house and subcontracted labor. The remaining definitions are those of Table 8.

B Theory Appendix

B.1 Stylized model

A risk-neutral firm is characterized by its stochastic present demand d , assumed to be Markovian with continuous cumulative distribution function (CDF) $G(d' | d)$. The economy demands up to a fixed quantity of the firm's good at given prices. Any difference between what is produced by the firm and what the economy wants becomes unmet demand. Production is linear in total input $f(x) = x$, which is the sum of in-house input and subcontracted production services $x = h + s$. Both inputs are obtained in competitive markets every period, at prices p_h and p_s , respectively. The firm's output is the numeraire, and it is assumed that $p_h < p_s < 1$.⁶⁶ In-house input is subject to time-to-build: next period's input must be chosen at the end of the current period, before future demand is realized. Conversely, subcontracting is chosen in the present period, allowing the firm to adjust its production upwards.

The value of the firm with current demand d is given by

$$V(d, h) = \max_{s \geq 0, h'} \left\{ \min\{d, h + s\} - p_h h - p_s s + \beta \mathbb{E}_{d'|d}[V(d', h')] \right\},$$

where β is the discount factor. The optimality condition for subcontracting tops up production up to the point where all demand is satisfied:

$$s^* = \max\{0, d - h\}.$$

Substituting, the problem becomes

$$V(d, h) = \max_{h'} \left\{ d - p_h h - p_s \max\{0, d - h\} + \beta \mathbb{E}_{d'|d}[V(d', h')] \right\}.$$

Taking first-order and envelope conditions, in-house input usage is given by

$$p_h = p_s[1 - G(h'^* | d)].$$

The marginal benefit of an additional unit of in-house input will be the subcontracting price if production is below realized demand, and zero otherwise; the marginal cost is the in-house input price. Consequently, the firm increases its capacity until the probability of surpassing its demand equals the normalized price spread between in-house input and subcontracting.⁶⁷

To make progress, I assume an invertible CDF $G(d' | d)$. Given the illustrative purpose of this

⁶⁶The price ordering is required for the demand of all inputs to be strictly positive. This assumption is relaxed in the extended model, where prices are endogenous objects.

⁶⁷This is a standard result in inventory policy problems (Arrow, Harris and Marschak, 1951), also widely studied in the operations management literature as the 'newsvendor' model (Morse and Kimball, 1946).

section, I assume that $d' \mid d \sim \mathcal{N}(\rho d, \sigma^2)$.⁶⁸ In-house input usage becomes:

$$h'^* = \rho d + \sigma \Phi^{-1}\left(\frac{p_s - p_h}{p_s}\right),$$

where Φ is the standard normal distribution function. In-house input is increasing in current demand, the persistence of demand and the price of subcontracting relative to in-house input, and it is increasing in the volatility of demand innovations only when $p_s > 2p_h$.

Proposition 3. *Let $p_h < p_s < 1$, and $d' \sim \mathcal{N}(\rho d, \sigma^2)$, where the signal-to-noise ratio is large enough that the probability of negative demand realizations is negligible. Define $\theta_s := s(d, h) / (h + s(d, h))$. The following properties characterize the firm's use of subcontracting:*

- (i) *The subcontracting share is increasing in realized demand: for given h , $\partial \theta_s / \partial d \geq 0$.*
- (ii) *The expected subcontracting share is increasing in demand volatility: $\partial \mathbb{E}_{d'} \theta'_s / \partial \sigma > 0$.*
- (iii) *Subcontracting comoves positively with output: $\text{Cov}(d', s^*) \geq 0$.*

Proof. See Appendix B.2.

Proposition 3 shows that the simple framework developed in this section satisfies the empirical observations from Sections 2.3 and 2.4. Subcontracting is used as a contemporaneous margin of adjustment to large demand shocks, so larger firms in terms of sales also present the largest subcontracting expenditure shares. The expected share of subcontracting services increases with the variance of the demand process. A mean-preserving spread of future demand raises the expected frequency and size of subcontracting relative to total production. Last, the positive correlation between demand and subcontracting comes by assumption, as subcontracting increases linearly with quantity demanded above a given threshold. Trivially, this will be larger than the covariance with in-house inputs, which is zero by construction.

This section concludes with a comparison between the policy functions across firms with and without subcontracting.

Proposition 4. *Let $p_h < p_s < 1$. Define h'^*_{NS} as the optimal next period's in-house input choice for a firm without access to subcontracting. Then, $h'^*_{NS} > h'^*$.*

Proof. See Appendix B.3.

Proposition 4 states that access to subcontracting markets that are perfectly substitutable for in-house inputs depresses demand for such inputs. This is a consequence of subcontracting acting only as upward insurance. Because firms with access to subcontracting can correct their production under large shocks, they focus on minimizing the losses from facing low output demand.

⁶⁸I also assume that $\rho > 0$ and that the signal-to-noise ratio is large enough to make the density over negative values negligible. Working with a truncated normal instead distorts the clarity of exposition.

The modeling choices in this section have been deliberately restrictive to highlight the core insights of subcontracting as a margin of adjustment. The model in Section 3 embeds the problem of the firm in general equilibrium and relaxes these functional assumptions. Prices become equilibrium objects, and the timing advantage of subcontracting over in-house production is endogenized.

B.2 Proof of Proposition 3

Part (i). The subcontracting share is

$$\theta_s = \frac{\max\{0, d - h\}}{\max\{d, h\}}.$$

Evaluating by cases:

$$\theta_s = \begin{cases} 0 & \text{if } d \leq h, \\ \frac{d-h}{d} & \text{if } d > h. \end{cases}$$

Taking the derivative with respect to d by cases:

$$\frac{\partial \theta_s}{\partial d} = \begin{cases} 0 & \text{if } d \leq h, \\ \frac{h}{d^2} > 0 & \text{if } d > h. \end{cases}$$

Therefore $\partial \theta_s / \partial d \geq 0$.

Part (ii). At the optimum, $h'^* = \mu + \sigma\alpha$, with $\mu = \rho d$ and φ the standard normal density, where $\alpha = \Phi^{-1}((p_s - p_h)/p_s)$ depends only on prices. The expected subcontracting share is

$$\mathbb{E}_{d'} \theta'_s = \int_{\alpha}^{\infty} \frac{\sigma z + \mu - h'^*}{\sigma z + \mu} \varphi(z) dz = \int_{\alpha}^{\infty} \frac{\sigma(z - \alpha)}{\sigma z + \mu} \varphi(z) dz.$$

Taking derivatives:

$$\frac{\partial}{\partial \sigma} \mathbb{E}_{d'} \theta'_s = \int_{\alpha}^{\infty} \frac{(z - \alpha)(\sigma z + \mu) - \sigma z(z - \alpha)}{(\sigma z + \mu)^2} \varphi(z) dz = \int_{\alpha}^{\infty} \frac{\mu(z - \alpha)}{(\sigma z + \mu)^2} \varphi(z) dz > 0,$$

where the inequality holds since $z > \alpha$ throughout the region of integration and $\mu > 0$.

Part (iii). Note that $\text{Cov}(d', s^*) = \mathbb{E}[d' s^*] - \mathbb{E}[d'] \mathbb{E}[s^*]$. I compute each term in turn.

Term 1: $\mathbb{E}[s^*]$. Note that $\alpha = \Phi^{-1}((p_s - p_h)/p_s) = (h'^* - \mu)/\sigma$ as defined in Part (ii). Since $s^* = \max\{0, d' - h'\} = \sigma \max\{0, z - \alpha\}$ where $z = (d' - \mu)/\sigma$, the expectation is the mean of a truncated normal:

$$\mathbb{E}[s^*] = \sigma \int_{\alpha}^{\infty} (z - \alpha) \varphi(z) dz = \sigma [\varphi(\alpha) - \alpha(1 - \Phi(\alpha))].$$

Therefore:

$$\mathbb{E}[d']\mathbb{E}[s^*] = \sigma\mu [\varphi(\alpha) - \alpha(1 - \Phi(\alpha))].$$

Term 2: $\mathbb{E}[d's^*]$. Since s^* is zero for $d' \leq h'$:

$$\mathbb{E}[d's^*] = \int_{\alpha}^{\infty} (\sigma z + \mu) \cdot \sigma(z - \alpha) \varphi(z) dz = \sigma^2 \int_{\alpha}^{\infty} z(z - \alpha) \varphi(z) dz + \sigma\mu \int_{\alpha}^{\infty} (z - \alpha) \varphi(z) dz.$$

For the first integral, using $\int_{\alpha}^{\infty} z^2 \varphi(z) dz = \alpha\varphi(\alpha) + (1 - \Phi(\alpha))$:

$$\int_{\alpha}^{\infty} z(z - \alpha) \varphi(z) dz = \int_{\alpha}^{\infty} z^2 \varphi(z) dz - \alpha \int_{\alpha}^{\infty} z \varphi(z) dz = [\alpha\varphi(\alpha) + (1 - \Phi(\alpha))] - \alpha\varphi(\alpha) = 1 - \Phi(\alpha).$$

The second integral was already computed in Term 1. Therefore:

$$\mathbb{E}[d's^*] = \sigma^2(1 - \Phi(\alpha)) + \sigma\mu [\varphi(\alpha) - \alpha(1 - \Phi(\alpha))].$$

Subtracting:

$$\text{Cov}(d', s^*) = \sigma^2(1 - \Phi(\alpha)) \geq 0.$$

□

B.3 Proof of Proposition 4

The firm without subcontracting access solves

$$V_{NS}(d, h) = \max_{h'_{NS}} \left\{ \min\{d, h\} - p_h h + \beta \mathbb{E}_{d'|d} [V_{NS}(d', h'_{NS})] \right\}.$$

In-house input demand is given by

$$p_h = 1 - G(h'^*_{NS} | d).$$

The condition $p_s < 1$ implies that $h'^*_{NS} > h'^*$.

□

B.4 Production function from sequential tasks

Consider different vertically connected stations $k = 1, \dots, N$ along a unit-length segment, each covering $\Delta = 1/N$ of the segment. Station k operates a Cobb–Douglas aggregator over the good it

inherits and effective labor, with labor elasticity $b\Delta$:

$$q_k = q_{k-1}^{1-b\Delta} (A_k \ell_k)^{b\Delta}, \quad q_0 = m,$$

where A_k is station k 's labor efficiency and the beginning of the chain is populated with materials m . Applying logs to both sides and iterating backwards, output can be expressed as

$$\ln q_N = (1 - b\Delta)^N \ln m + b\Delta \sum_{k=1}^N (1 - b\Delta)^{N-k} \ln(A_k \ell_k).$$

Index stations by their location $s_k = k/N$ and let $A_k = A(s_k)$, $\ell_k = \ell(s_k)$. Using $\lim_{\Delta \rightarrow 0} N \ln(1 - b\Delta) = -b$, and so $\lim_{\Delta \rightarrow 0} (1 - b\Delta)^{N(1-s)} = e^{-b(1-s)}$, I express the above summation as a Riemann sum with mesh Δ and take limits:

$$b\Delta \sum_{k=1}^N (1 - b\Delta)^{N-k} \ln(A_k \ell_k) = \sum_{k=1}^N b (1 - b\Delta)^{N(1-s_k)} \ln(A(s_k) \ell(s_k)) \Delta \longrightarrow \int_0^1 b e^{-b(1-s)} \ln(A(s) \ell(s)) ds.$$

Applying this and $(1 - b\Delta)^N \rightarrow e^{-b}$ to the expression above yields equation (2).

B.5 Proof of Lemma 1

Define the change of variables from physical line position s to task weights u : $u(s) = 1 - e^{-b(1-s)}$, $s(u) = 1 + \ln(1 - u)/b$, with $du = -\zeta(s)ds$. Define in weight units the task efficiency $\tilde{A}_{it}(u) = A_{it}(s(u))$, and the labor density such that it preserves its physical allocation $\tilde{\ell}_{it}(u) = \ell_{it}(s(u))/\zeta(s(u))$.⁶⁹ Applying the change of variables to the labor component of equation (2),

$$\int_0^1 \zeta(s) \ln(A_{it}(s) \ell_{it}(s)) ds = \int_0^{1-\alpha_m} \ln(\tilde{A}_{it}(u) \tilde{\ell}_{it}(u)) du + \underbrace{\int_0^{1-\alpha_m} \ln \zeta(s(u)) du}_{=\int_0^1 \zeta(s) \ln(\zeta(s)) ds \equiv \mathcal{K}},$$

which equals equation (4). □

B.6 Proof of Corollary 1

Evaluating $u(s)$ at the key points:

$$u(0) = 1 - \alpha_m, \quad u(1) = 0, \quad u(\bar{s}_i) = 1 - e^{-b(1-\bar{s}_i)} \equiv \bar{\mu}_i, \quad u(\hat{s}_{it}) = 1 - e^{-b(1-\hat{s}_{it})} \equiv \hat{\mu}_{it}.$$

Since $u(s)$ is strictly decreasing in s , clauses (i) and (ii) follow.

⁶⁹A naive definition $\tilde{\ell}(u) = \ell(s(u))$ would not preserve total labor over u and thus lack any physical meaning: $\int_0^{1-\alpha_m} \tilde{\ell}_{it}(u) du = \int_0^1 \zeta(s) \ell_{it}(s) ds \neq \int_0^1 \ell_{it}(s) ds = l_{it}$.

Because $s(u)$ is strictly decreasing and $\gamma(s; \bar{s}_i)$ is strictly increasing in s , $\tilde{\gamma}(u; \bar{\mu}_i) = \gamma(s(u); \bar{s}_i)$ is strictly decreasing in u on $[0, \bar{\mu}_i]$, proving clause (iii). $\square$

B.7 Firm equilibrium conditions

This section solves the problem of a firm with access to subcontracting services. The optimality conditions for a firm without access, namely the materials and in-house labor cost shares, follow from imposing zero delegation $\hat{\mu} = 0$ in the problem with access.

B.7.1 Uniform labor spread across tasks

Take the problem of a firm with access with the technology described by equation (4):

$$V_S(z, \varepsilon, l_{H,-1}, \bar{\mu}) = \max_{\tilde{\ell}_H(u), m, \hat{\mu} \in [0, \bar{\mu}], \tilde{\ell}_S(u)} \left\{ z\varepsilon \left[m^{\alpha_m} e^{\int_{\hat{\mu}}^{1-\alpha_m} \ln \tilde{\ell}_H(u) du + \int_0^{\hat{\mu}} \ln \tilde{\ell}_S(u) du + \int_0^{\hat{\mu}} \ln(\phi \tilde{\gamma}(u; \bar{\mu})) du} \right]^\nu \right. \\ \left. - w l_H - c(l_H, l_{H,-1}) - \omega l_S - m - \kappa_F \right. \\ \left. + \beta \left[(1 - \delta_X) \max \left\{ -c(0, l_H), \mathcal{V}_S(z, l_H, \bar{\mu}) \right\} - \delta_X c(0, l_H) \right] \right\},$$

where $l_H = \int_{\hat{\mu}}^{1-\alpha_m} \tilde{\ell}_H(u) du$, $l_S = \int_0^{\hat{\mu}} \tilde{\ell}_S(u) du$, and I have used the normalization $e^{\mathcal{K}} A_0^{1-\alpha_m} = 1$. The FOCs for in-house labor and subcontracting intensity are

$$\frac{vy}{\tilde{\ell}_H(u)} = \tilde{w} \quad \forall u \in [\hat{\mu}, 1 - \alpha_m], \quad \frac{vy}{\tilde{\ell}_S(u)} = \omega \quad \forall u \in [0, \hat{\mu}).$$

Since optimal labor intensities are constant across tasks, $\tilde{\ell}_H(u) = \tilde{\ell}_H \forall u \in [\hat{\mu}, 1 - \alpha_m]$, $\tilde{\ell}_S(u) = \tilde{\ell}_S \forall u \in [0, \hat{\mu})$, the problem can be reduced to choosing the total amount of in-house and subcontracted labor for production.

Integrating $\int_{\hat{\mu}}^{1-\alpha_m} \ln \tilde{\ell}_H(u) du = (1 - \alpha_m - \hat{\mu}) \ln \tilde{\ell}_H$, $\int_0^{\hat{\mu}} \ln \tilde{\ell}_S(u) du = \hat{\mu} \ln \tilde{\ell}_S$, and $\int_0^{\hat{\mu}} \ln(\phi \tilde{\gamma}(u; \bar{\mu})) du = \hat{\mu} \ln \phi + \ln B(\hat{\mu})$, using that at the optimum $\tilde{\ell}_H = l_H / (1 - \alpha_m - \hat{\mu})$ and $\tilde{\ell}_S = l_S / \hat{\mu}$, and substituting into the technology, I obtain the expression in equation (10). Taking the limit $\hat{\mu} \rightarrow 0$ drops the delegated block, giving (8).

B.7.2 Proof of Proposition 1

The first-order conditions of problem (10) with respect to variable inputs give the standard Cobb–Douglas input shares:

$$m = \alpha_m vy, \quad \omega l_S = \hat{\mu} vy, \quad \tilde{w} l_H = (1 - \alpha_m - \hat{\mu}) vy.$$

Given l_H and l_S , the delegated weight $\hat{\mu}$ affects only revenue, so the condition for $\hat{\mu}$ is $\partial y / \partial \hat{\mu} = 0$. Differentiating,

$$\frac{1}{vy} \frac{\partial y}{\partial \hat{\mu}} = \ln \tilde{\gamma}(\hat{\mu}; \bar{\mu}) + \left[\ln \frac{\phi l_S}{\hat{\mu}} - 1 \right] + \left[-\ln \frac{l_H}{1 - \alpha_m - \hat{\mu}} + 1 \right] = 0,$$

leaving

$$\tilde{\gamma}(\hat{\mu}; \bar{\mu}) \frac{\phi l_S / \hat{\mu}}{l_H / (1 - \alpha_m - \hat{\mu})} = 1.$$

Substituting the subcontracting and in-house labor cost shares, $l_S / \hat{\mu} = vy / \omega$ and $l_H / (1 - \alpha_m - \hat{\mu}) = vy / \tilde{w}$, delivers the interior optimal delegated weight

$$\phi \tilde{\gamma}(\hat{\mu}; \bar{\mu}) = \frac{\omega}{\tilde{w}},$$

which is the solution whenever the implied $\hat{\mu}$ is interior, and at a corner otherwise. $\square$

B.7.3 Proof of Proposition 2

Apply logarithms to g and substitute the optimality conditions in (12):

$$\begin{aligned} \ln g &= \int_0^{\hat{\mu}} \ln \tilde{\gamma}(u; \bar{\mu}) du + \alpha_m \ln m + \hat{\mu} \ln(\phi l_S / \hat{\mu}) + (1 - \alpha_m - \hat{\mu}) \left(\ln l_H - \ln(1 - \alpha_m - \hat{\mu}) \right) \\ &= \int_0^{\hat{\mu}} \ln \tilde{\gamma}(u; \bar{\mu}) du + \alpha_m (\ln \alpha_m + \ln vy) + \hat{\mu} (\ln \phi + \ln vy - \ln \omega) + (1 - \alpha_m - \hat{\mu}) (\ln vy - \ln \tilde{w}) \\ &= \int_0^{\hat{\mu}} \ln \tilde{\gamma}(u; \bar{\mu}) du + \ln vy + \alpha_m \ln \alpha_m + \hat{\mu} (\ln \phi - \ln \omega) - (1 - \alpha_m - \hat{\mu}) \ln \tilde{w}. \end{aligned}$$

Rearranging terms and substituting the interior equilibrium condition (11):

$$\begin{aligned} \ln \frac{vy}{g} &= -\alpha_m \ln \alpha_m - \int_0^{\hat{\mu}} \ln \tilde{\gamma}(u; \bar{\mu}) du + \hat{\mu} (\ln \omega - \ln \phi) + (1 - \alpha_m - \hat{\mu}) \ln \tilde{w} \quad (\text{B.1}) \\ &= -\alpha_m \ln \alpha_m - \int_0^{\hat{\mu}} \ln \tilde{\gamma}(u; \bar{\mu}) du + \hat{\mu} \ln \tilde{\gamma}(\hat{\mu}; \bar{\mu}) + (1 - \alpha_m) \ln \tilde{w} \\ &= -\alpha_m \ln \alpha_m - \int_0^{\hat{\mu}} \ln \frac{\tilde{\gamma}(u; \bar{\mu})}{\tilde{\gamma}(\hat{\mu}; \bar{\mu})} du + (1 - \alpha_m) \ln \tilde{w} \\ \Rightarrow \frac{vy}{g} &= \alpha_m^{-\alpha_m} e^{-\int_0^{\hat{\mu}} \ln \frac{\tilde{\gamma}(u; \bar{\mu})}{\tilde{\gamma}(\hat{\mu}; \bar{\mu})} du} \tilde{w}^{1-\alpha_m} \end{aligned}$$

From the cost shares in (12), total variable costs evaluated at shadow prices are vy , hence the unit

cost is $c_g \equiv \nu y/g$. Substituting the delegation gains $R(\hat{\mu})$ of Proposition 2, I get

$$c_g = \alpha_m^{-\alpha_m} e^{-R(\hat{\mu})} \tilde{w}^{1-\alpha_m}.$$

At a corner, condition (11) does not hold, and the first line of (B.1) can instead be written as

$$\ln \frac{\nu y}{g} = -\alpha_m \ln \alpha_m - \int_0^{\hat{\mu}} \ln \frac{\phi \tilde{\gamma}(u; \bar{\mu}) \tilde{w}}{\omega} du + (1 - \alpha_m) \ln \tilde{w},$$

which gives the same expression for c_g with $R(\hat{\mu})$ replaced by $R_c(\hat{\mu})$. Moreover, since $\tilde{\gamma}(u; \bar{\mu})$ is strictly decreasing in u , $\tilde{\gamma}(u; \bar{\mu}) \geq \tilde{\gamma}(\hat{\mu}; \bar{\mu})$ for every $u \leq \hat{\mu}$, so $R(\hat{\mu}) \geq 0$.

Last, taking derivatives of equation (B.1) with respect to $\tilde{w}$ and using the envelope theorem gives

$$\begin{aligned} \frac{\partial \ln c_g}{\partial \ln \tilde{w}} &= 1 - \alpha_m - \hat{\mu} + \underbrace{\left[-\ln \tilde{\gamma}(\hat{\mu}; \bar{\mu}) + (\ln \omega - \ln \phi) - \ln \tilde{w} \right]}_{=0} \frac{d\hat{\mu}}{d \ln \tilde{w}} \\ &= 1 - \alpha_m - \hat{\mu}, \end{aligned}$$

which is the passthrough of the shadow cost of in-house labor to unit costs. $\square$

B.8 Specialization gains as coordination savings

This section shows that the problem of a firm with access is isomorphic to one in which the subcontractor performs every delegable task with the same efficiency, and coordinating in-house production requires more labor the more tasks the firm performs itself.

Assume the subcontractor performs any delegated task with stage productivity $A_0 \xi$, where $\xi > 0$ is common to all tasks, and that tasks performed in-house have stage productivity A_0 . Performing tasks in-house also requires coordination (similarly to the overhead costs of spanning multiple products in Aghion, Bergeaud, Boppart, Klenow and Li, 2023). Each unit of production labor requires $\mathcal{C}(\hat{\mu}; \bar{\mu})$ in-house workers, so that the requirement decreases with the weight of delegated tasks. A firm that delegates nothing needs the same coordination no matter its type, so $\mathcal{C}(0; \bar{\mu}) = 1$. Let l_P denote production labor, so that the in-house workforce is $l_H = \mathcal{C}(\hat{\mu}; \bar{\mu}) l_P$. With labor spread evenly across tasks, as in Appendix B.7, the problem of a firm with access is

$$\begin{aligned} V_S(z, \varepsilon, l_{H,-1}, \bar{\mu}) &= \max_{l_P, m, \hat{\mu} \in [0, \bar{\mu}], l_S} \left\{ z \varepsilon \left[m^{\alpha_m} \left(\frac{\xi l_S}{\hat{\mu}} \right)^{\hat{\mu}} \left(\frac{l_P}{1 - \alpha_m - \hat{\mu}} \right)^{1 - \alpha_m - \hat{\mu}} \right]^\nu \right. \\ &\quad \left. - w l_H - c(l_H, l_{H,-1}) - \omega l_S - m - \kappa_F \right. \\ &\quad \left. + \beta \left[(1 - \delta_X) \max \left\{ -c(0, l_H), \mathcal{V}_S(z, l_H, \bar{\mu}) \right\} - \delta_X c(0, l_H) \right] \right\}, \end{aligned}$$

with $l_H = \mathcal{C}(\hat{\mu}; \bar{\mu}) l_P$.

The first-order conditions for materials, subcontracted labor and production labor are

$$m = \alpha_m \nu y, \quad \omega l_S = \hat{\mu} \nu y, \quad \tilde{w} C(\hat{\mu}; \bar{\mu}) l_P = (1 - \alpha_m - \hat{\mu}) \nu y,$$

where $\tilde{w}$ is the shadow cost of in-house labor. Note that, since $\tilde{w} C(\hat{\mu}; \bar{\mu}) l_P = \tilde{w} l_H$, these are the cost shares in (12). The condition for the delegated weight is

$$\frac{\partial y}{\partial \hat{\mu}} = \tilde{w} \frac{\partial C(\hat{\mu}; \bar{\mu})}{\partial \hat{\mu}} l_P.$$

Dividing by νy and using the cost share of production labor,

$$\left[\ln \frac{\xi l_S}{\hat{\mu}} - 1 \right] + \left[-\ln \frac{l_P}{1 - \alpha_m - \hat{\mu}} + 1 \right] = (1 - \alpha_m - \hat{\mu}) \frac{\partial \ln C(\hat{\mu}; \bar{\mu})}{\partial \hat{\mu}}.$$

Substituting $l_S/\hat{\mu} = \nu y/\omega$ and $l_P/(1 - \alpha_m - \hat{\mu}) = \nu y/(\tilde{w} C(\hat{\mu}; \bar{\mu}))$ delivers the interior delegated weight

$$\ln(\xi C(\hat{\mu}; \bar{\mu})) - (1 - \alpha_m - \hat{\mu}) \frac{\partial \ln C(\hat{\mu}; \bar{\mu})}{\partial \hat{\mu}} = \ln \frac{\omega}{\tilde{w}}.$$

The first term is the log ratio between the output of a worker at the marginal task when delegated, ξ , and when it is performed in-house, $1/C(\hat{\mu}; \bar{\mu})$. The second term is the coordination saved on the tasks that remain in-house.

Since Proposition 1 gives $\ln(\phi \tilde{\gamma}(\hat{\mu}; \bar{\mu})) = \ln(\omega/\tilde{w})$, the two problems deliver the same delegated weight if, for every type and every $\hat{\mu} \in [0, \bar{\mu}]$, the following condition holds:

$$\ln(\phi \tilde{\gamma}(\hat{\mu}; \bar{\mu})) = \ln(\xi C(\hat{\mu}; \bar{\mu})) - (1 - \alpha_m - \hat{\mu}) \frac{\partial \ln C(\hat{\mu}; \bar{\mu})}{\partial \hat{\mu}}.$$

Integrating both sides of this condition from 0 to $\hat{\mu}$, and using the initial conditions $B(0) = 1$ and $C(0; \bar{\mu}) = 1$, gives

$$\hat{\mu} \ln \phi + \ln B(\hat{\mu}) = \hat{\mu} \ln \xi - (1 - \alpha_m - \hat{\mu}) \ln C(\hat{\mu}; \bar{\mu}).$$

The left-hand side is the log of $\phi^{\hat{\mu}} B(\hat{\mu})$, the productivity shifter that specialization adds to the technology in (10). The right-hand side is also the log of a productivity shifter. Writing production labor as $l_P = l_H/C(\hat{\mu}; \bar{\mu})$, output in the alternative technology is

$$g = \xi^{\hat{\mu}} C(\hat{\mu}; \bar{\mu})^{-(1 - \alpha_m - \hat{\mu})} m^{\alpha_m} \left(\frac{l_S}{\hat{\mu}} \right)^{\hat{\mu}} \left(\frac{l_H}{1 - \alpha_m - \hat{\mu}} \right)^{1 - \alpha_m - \hat{\mu}},$$

the same combination of inputs as in (10) multiplied by $\xi^{\hat{\mu}} C(\hat{\mu}; \bar{\mu})^{-(1 - \alpha_m - \hat{\mu})}$. Both technologies

therefore produce the same output with the same inputs and deliver the same delegated weight, so the two problems are isomorphic. $\square$

B.9 Aggregation

B.9.1 Productivity

Let $Y_t = \int_i y_{it} di$, $G_t = \int_i g_{it} di$, $M_t = \int_i m_{it} di$, $\mathcal{L}_t = \int_i (l_{H,it} + l_{S,it}) di$, $\Lambda_t = \int_i di$, and $\bar{g}_t = G_t / \Lambda_t$. Aggregate productivity is

$$Z_t = \frac{Y_t}{\Lambda_t^{1-\nu} \left[M_t^{\alpha_m} \left(\frac{\mathcal{L}_t}{1-\alpha_m} \right)^{1-\alpha_m} \right]^\nu}.$$

Note the numerator can be written as $Y_t = \int_i z_{it} \varepsilon_{it} g_{it}^\nu di = G_t^\nu \int_i z_{it} \varepsilon_{it} \sigma_{G,it}^\nu di$, where $\sigma_{G,it} = g_{it} / G_t$, which collects the distribution of idiosyncratic productivities across firms. Consequently, I can write

$$\Lambda_t^{1-\nu} Z_t = \left(\int_i z_{it} \varepsilon_{it} \sigma_{G,it}^\nu di \right) \underbrace{\left[\frac{G_t}{M_t^{\alpha_m} \left(\frac{\mathcal{L}_t}{1-\alpha_m} \right)^{1-\alpha_m}} \right]^\nu}_{\equiv \mathcal{A}_t} \quad (\text{B.2})$$

The first factor can be separated into the size of the economy and size-weighted productivity, P_t . Since $\sigma_{G,it} = g_{it} / G_t = \Lambda_t^{-1} g_{it} / \bar{g}_t$,

$$\int_i z_{it} \varepsilon_{it} \sigma_{G,it}^\nu di = \Lambda_t^{-\nu} \int_i z_{it} \varepsilon_{it} \left(\frac{g_{it}}{\bar{g}_t} \right)^\nu di = \Lambda_t^{1-\nu} \mathbb{E}_t \left[z_{it} \varepsilon_{it} \left(\frac{g_{it}}{\bar{g}_t} \right)^\nu \right] = \Lambda_t^{1-\nu} P_t,$$

where $\mathbb{E}_t$ denotes the unweighted cross-sectional average.

Recall from the optimal input expenditure shares in (12) that in-house labor intensity satisfies $\tilde{\ell}_{H,it} = l_{H,it} / (1 - \alpha_m - \hat{\mu}_{it}) = \nu y_{it} / \tilde{w}_{it}$. Substitute it into the definition for aggregate materials to obtain

$$M_t = \alpha_m \int_i \nu y_{it} di = \alpha_m \int_i \tilde{w}_{it} \tilde{\ell}_{H,it} di.$$

I can also express total labor as $l_{H,it} + l_{S,it} = [(1 - \alpha_m - \hat{\mu}_{it}) + \hat{\mu}_{it} \tilde{w}_{it} / \omega_t] \tilde{\ell}_{H,it} = (1 - \alpha_m) h_{it} \tilde{\ell}_{H,it}$, where $h_{it} = [(1 - \alpha_m - \hat{\mu}_{it}) + \hat{\mu}_{it} \tilde{w}_{it} / \omega_t] / (1 - \alpha_m)$, and total aggregate labor as

$$\mathcal{L}_t = (1 - \alpha_m) \int_i h_{it} \tilde{\ell}_{H,it} di.$$

Last, using the unit cost expression in (13), with $R_c(\hat{\mu}_{it})$ in place of $R(\hat{\mu}_{it})$ for firms at a corner,

$$G_t = \int_i \frac{\nu y_{it}}{c_{g,it}} di = \alpha_m^{\alpha_m} \int_i e^{R(\hat{\mu}_{it})} \tilde{w}_{it}^{\alpha_m} \tilde{\ell}_{H,it} di.$$

Substitute all terms into the expression for $\mathcal{A}_t$ to obtain equation (17):

$$\mathcal{A}_t = \frac{G_t}{M_t^{\alpha_m} \left(\frac{\mathcal{L}_t}{1-\alpha_m} \right)^{1-\alpha_m}} = \frac{\int_i e^{R(\hat{\mu}_{it})} \tilde{w}_{it}^{\alpha_m} \tilde{\ell}_{H,it} di}{\left(\int_i \tilde{w}_{it} \tilde{\ell}_{H,it} di \right)^{\alpha_m} \left(\int_i h_{it} \tilde{\ell}_{H,it} di \right)^{1-\alpha_m}} = \frac{\mathbb{E}_\varsigma [e^{R(\hat{\mu}_{it})} \tilde{w}_{it}^{\alpha_m}]}{(\mathbb{E}_\varsigma [\tilde{w}_{it}])^{\alpha_m} (\mathbb{E}_\varsigma [h_{it}])^{1-\alpha_m}},$$

with $R_c(\hat{\mu}_{it})$ in place of $R(\hat{\mu}_{it})$ for firms at a corner, where $\mathbb{E}_\varsigma$ denotes the cross-sectional average with weights $\varsigma_{it} = \tilde{\ell}_{H,it} / \int_i \tilde{\ell}_{H,it} di$. Substituting into expression (B.2) and dividing by $\Lambda_t^{1-\nu}$ gives $Z_t = P_t \mathcal{A}_t^\nu$.

B.9.2 Aggregate labor wedge

Define the aggregate labor wedge $\tau_{L,t}$ as a tax on the price of labor that rationalizes the aggregate labor allocation:

$$\nu(1-\alpha_m) \frac{Y_t}{\mathcal{L}_t} = \tau_{L,t} w_t.$$

From the optimality conditions in (12) write $(1-\alpha_m)\nu y_{it} = (1-\alpha_m-\hat{\mu}_{it})\nu y_{it} + \hat{\mu}_{it}\nu y_{it} = \tilde{w}_{it} l_{H,it} + \omega_t l_{S,it}$, and integrating over firms I obtain

$$\nu(1-\alpha_m)Y_t = \nu(1-\alpha_m) \int_i y_{it} di = \int_i \tilde{w}_{it} l_{H,it} di + \omega_t \int_i l_{S,it} di = \bar{w}_t L_{H,t} + \omega_t L_{S,t},$$

where $\bar{w}_t = \int_i \tilde{w}_{it} l_{H,it} / L_{H,t} di$. Define the share of total in-house labor $s_{H,t} = L_{H,t} / \mathcal{L}_t$ to obtain the aggregate wedge

$$\tau_{L,t} = s_{H,t} \frac{\bar{w}_t}{w_t} + (1-s_{H,t}) \frac{\omega_t}{w_t},$$

which is (19).

B.10 Transitional dynamics under aggregate shocks

The economy is at the stationary equilibrium described in Section 3. At the beginning of $t = 0$, aggregate productivity changes unexpectedly and follows the path $A_t = \exp(\Delta_A \rho_A^t)$ thereafter, where Δ_A is the size of the shock and ρ_A its persistence. A_t multiplies the revenue of every firm at t . Agents learn the entire path when the shock is revealed (perfect foresight). Entry and access efforts at $t = 0$ are chosen before the shock is revealed, so they take their stationary values.

Satisfying the free-entry condition (16) at every date generates large swings in entry that distort the response of the economy. A positive shock raises the value of entering, so the wage must rise to lower it back to κ_E . The higher wage raises labor supply, which incumbents cannot absorb because of adjustment frictions, so the mass of entrants must rise to clear the labor market. The following period, the mass of incumbents and labor demand are large, so entry must fall to satisfy labor market clearing and free entry. Thus, entry oscillates with a large amplitude, and consumption and output inherit these swings.

For this exercise, I work with an entry margin that delivers a smoother aggregate response. In particular, I assume that there is a finite mass $\tilde{E}$ of potential entrants that are heterogeneous in their entry cost. Each draws a cost κ from the distribution $F_\kappa(\kappa) = (\kappa/\bar{\kappa})^2$ on $[0, \bar{\kappa}]$ and enters if the value of entering covers it. The mass of entrants at t is then

$$E_t = \tilde{E} F_\kappa(V_{E,t}), \quad \text{where}$$

$$V_{E,t} = \int \max_{e \geq 0} \left\{ -w_t e + p(e) \mathbb{E}_{\hat{G}}[V_{S,t}(z, \varepsilon, 0, \bar{\mu})] + (1 - p(e)) \mathbb{E}_{\hat{G}}[V_{NS,t}(z, \varepsilon, 0, \bar{\mu})] \right\} dF_{\bar{\mu}}(\bar{\mu}),$$

so entry responds to the value of entering with an elasticity of 2.^{70,71} I set $\tilde{E}$ and $\bar{\kappa}$ such that the marginal entrant's cost at the stationary mass of entrants equals κ_E . The stationary equilibrium is then the same as under condition (16), and only the response of entry off the steady state changes.

Because the household's utility is linear in consumption, firms and the subcontractor discount the future at β along the transition.

Given the stationary equilibrium at $t = -1$ and the path $\{A_t\}$, a transition is a sequence of allocations $\{C_t, N_t\}$, prices $\{w_t, \omega_t\}$, value functions and policies, entry masses $\{E_t\}$, and distributions of firms $\{\lambda_t\}$ such that:

1. Firms and the subcontractor optimize given the price paths.
2. The price of subcontracting satisfies the subcontractor's pricing condition (7) at every date.
3. The allocations maximize household utility given prices, with labor supply and consumption satisfying equations (6) and (5) at every date.
4. Entry satisfies the condition above at every date.
5. The labor market clears at every date, and by Walras' law so does the goods market.
6. The distribution of firms evolves according to the firms' policies and the idiosyncratic shocks to productivity and access.
7. The economy is back at the steady state from the terminal date T onwards.

⁷⁰Inframarginal entrants earn rents, which accrue to the household.

⁷¹The comparisons between economies are not sensitive to this choice, and hold with elasticities of 1 and 4.

The subcontractor's profits and losses along the transition accrue to the household as a lump sum.

B.11 The value of subcontracting under transitory and persistent shocks

This subsection studies the value of subcontracting access across shocks of different persistence. A persistent shock delivers a lasting change in scale, for which in-house adjustment can be worth it, as subcontracting the new scale permanently would imply delegating less codifiable tasks and lowering the effective yield per task. Conversely, a transitory shock is unlikely to justify costly adjustment, and firms might prefer to absorb it through subcontracting. I test this distinction quantitatively to better understand where the value of subcontracting resides.

I begin by defining a measure of equivalence across shocks. For a transitory shock of size x_ε , I find the persistent innovation $x_z(x_\varepsilon, z, l_{H,-1}, \bar{\mu})$ such that a firm without subcontracting access values them identically. This is implicitly defined by

$$V_{NS}(ze^{x_z}, 1, l_{H,-1}, \bar{\mu}) = V_{NS}(z, e^{x_\varepsilon}, l_{H,-1}, \bar{\mu}).$$

In order to measure the value of subcontracting access under a transitory or a persistent shock, I define the access premia

$$\begin{aligned}\mathcal{P}_\varepsilon(x_\varepsilon, z, l_{H,-1}, \bar{\mu}) &= V_S(z, e^{x_\varepsilon}, l_{H,-1}, \bar{\mu}) - V_{NS}(z, e^{x_\varepsilon}, l_{H,-1}, \bar{\mu}), \\ \mathcal{P}_z(x_\varepsilon, z, l_{H,-1}, \bar{\mu}) &= V_S(ze^{x_z}, 1, l_{H,-1}, \bar{\mu}) - V_{NS}(ze^{x_z}, 1, l_{H,-1}, \bar{\mu}).\end{aligned}$$

Then, I define the transitory premium, the incremental value of subcontracting access under a transitory shock over an equivalent persistent one, as

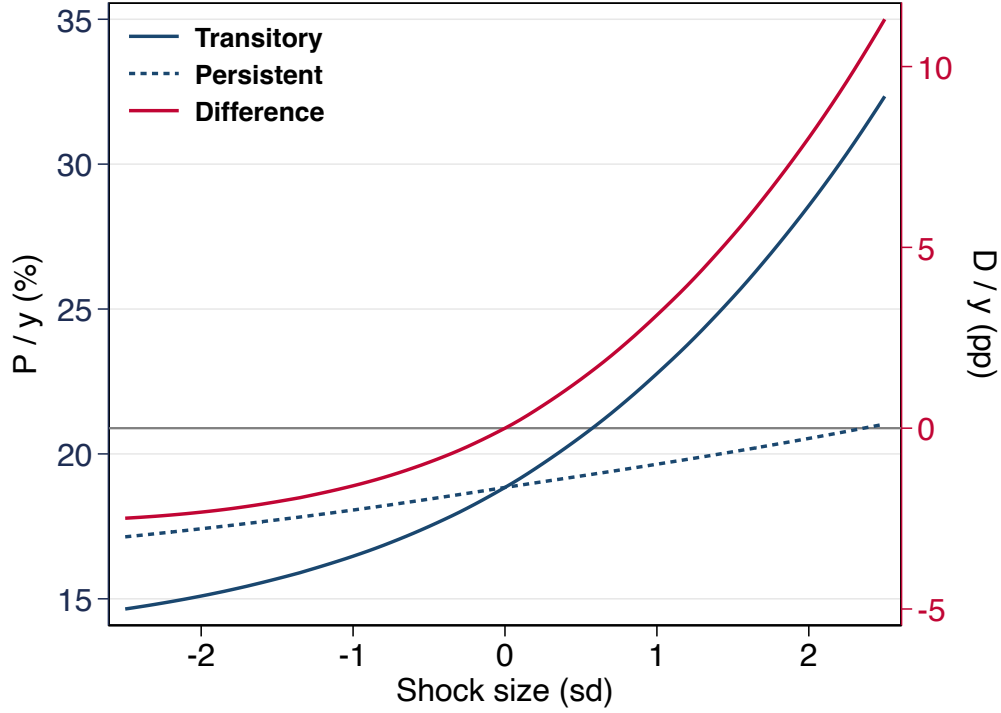
$$\begin{aligned}\mathcal{D}_{\varepsilon z}(x_\varepsilon, z, l_{H,-1}, \bar{\mu}) &= \mathcal{P}_\varepsilon(x_\varepsilon, z, l_{H,-1}, \bar{\mu}) - \mathcal{P}_z(x_\varepsilon, z, l_{H,-1}, \bar{\mu}) \\ &= V_S(z, e^{x_\varepsilon}, l_{H,-1}, \bar{\mu}) - V_S(ze^{x_z}, 1, l_{H,-1}, \bar{\mu}).\end{aligned}$$

Because the no-access values coincide by construction, this difference measures how the value of access compares across shock types.

Figure B.1 shows the average value of subcontracting access under each type of shock, and the average transitory premium, for transitory shocks of different magnitude.⁷² The average value of subcontracting access under persistent shocks is relatively flat, ranging from 17% to 21% of the firm's yearly sales for shocks between two standard deviations below and above the mean. Conversely, having access to subcontracting under a transitory shock shows an increasing value along the shock size axis: from 15% of yearly sales for a negative shock of two standard deviations below the mean, to 29%

⁷²I average these objects over the firms in the stationary distribution, at the persistent productivity each firm would have without a new innovation. I normalize them relative to the sales the firm would have with access in the absence of the shock, to provide a meaningful monetary value.

Figure B.1: The value of subcontracting under transitory and persistent shocks.



Notes: The figure plots the average access premia $\mathcal{P}_\varepsilon$ and $\mathcal{P}_z$ (left axis), and their difference $\mathcal{D}_{\varepsilon z}$ (right axis), against the size of the transitory shock, in standard deviations of $\ln \varepsilon$. All objects are expressed relative to the firm's sales with access in the absence of the shock, and averaged across firms.

for a shock of the same size but positive. Access is most valuable when a firm needs to expand only temporarily, and less so when a transitory contraction leaves excess in-house capacity. The transitory premium also increases with the size of the shock, from -2.3 to 8.0 percentage points of yearly sales over the same range.

These results confirm the distinction that motivates this subsection. The value of subcontracting is largest against transitory expansions, where in-house adjustment is least worthwhile, and smallest against transitory contractions, which firms can absorb with the labor they already have. Persistent shocks, in contrast, change the value of access much less, as firms can meet them by adjusting in-house. The flexibility motive is thus a demand for absorbing temporary movements.

Appendix References

- Aghion, Philippe, Antonin Bergeaud, Timo Boppart, Peter J Klenow, and Huiyu Li**, “A theory of falling growth and rising rents,” *The Review of Economic Studies*, 2023, 90 (6), 2675–2702.
- Arrow, Kenneth J, Theodore Harris, and Jacob Marschak**, “Optimal inventory policy,” *Econometrica*, 1951, pp. 250–272.
- Bontadini, F., C. Corrado, J. Haskel, M. Iommi, and C. Jona-Lasinio**, “EUKLEMS & INTANProd: Industry Productivity Accounts with Intangibles. Sources of Growth and Productivity Trends: Methods and Main Measurement Challenges,” Deliverable D2.3.1, European Commission, Directorate-General for Economic and Financial Affairs February 2023. Procurement procedure ECFIN/2020/O.P./0001.
- Gouveia, Ana Fontoura and Manuel Coutinho Pereira**, “An Economic Estimate of Capital Stock at the Firm Level for Portugal,” Occasional Paper 04/2022, Banco de Portugal, Lisbon September 2022.
- Morse, Philip M and George E Kimball**, “Methods of operations research,” Technical Report 1946.